PERSONNEL INVENTORY PROJECTION, ENLISTED (PIPE)

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Thesis

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ABSTRACT

One of the responsibilities of the personnel planner of the Bureau of Naval Personnel is the projection of current personnel inventories into the future and the determination of the personnel replacements that are necessary in order for these inventories to meet stated requirements. The personnel planner presently has a method of solution to this problem no more sophisticated than hand computations, and consequently it is a long and arduous task.

The process by which a personnel inventory changes with the passage of time is suitable for programming on a digital computer. The complexities that this problem generates because it considers the behavior of people only serves to amplify the utility of a digital computer to perform this function in preference to hand computations.

This thesis is the development of the program PIPE, written in FORTRAN 60 for the 1604 digital computer, which will project current enlisted personnel inventories of certain subsystems of the Navy for as much as five years into the future, and determine the least expensive personnel replacements necessary to enable these projected inventories to meet stated requirements. The program reduces the solution time of this particular problem of the personnel planner to minutes.

PIPE is not designed to be a mechanical process to be used blindly. Rather, this program can be used effectively to improve

the art of personnel inventory projection when the personnel planner uses his experience to properly interpret the results.

An analysis of PIPE with respect to its current and future potential to the personnel planner led to certain conclusions and recommendations that are enumerated in this paper.

PREFACE

The Naval personnel planner is faced today with an increasingly complex problem: how to provide enough adequately trained enlisted men, on time, to meet the requirements of rapidly expanding new weapons systems.

The authors became interested in this problem during the course of temporary additional assignments in Washington, D. C., during the summer of 1963. These assignments were made as part of the Operations Analysis Curriculum at the U. S. Naval Post-graduate School. Lieutenant Commanders Meeks and Metcalf spent six weeks at the Bureau of Naval Personnel, and during this period became well acquainted with the Bureau's current methods used in meeting this problem, as well as with some new concepts that the Bureau was considering for use in this problem area. Lieutenants Greenhalgh and Harper were working at the Special Projects Office, and there became impressed with the concern shown for the difficulties involved in planning for sufficient enlisted personnel to meet the requirements of the Polaris program.

The four officer-students thought it to be beneficial to pool their experience and knowledge of the problem in a combined effort which would attempt to derive a simple yet useful program for the solution of this personnel problem. Upon closer examination it was

seen that the problem was large and complex enough to indeed merit the combined efforts of the four officers, and that such effort, if carried to a conclusion comensurate with their proposals, would satisfy the requirements for a thesis in the Master's Degree Program of the Operations Analysis Curriculum of the Naval Postgraduate School.

The result of this effort is Personnel Inventory Projection, Enlisted, (PIPE). PIPE consists of a linear programming model along with a digital computer program that will project personnel inventories and determine the least expensive personnel replacements that are necessary to enable these projected inventories to meet stated requirements.

The development of the model and the computer program has been subdivided into sections so as to allow readers with different interests to read only the pertinent sections of the thesis.

A detailed discussion of the problem of projecting personnel inventories and the reasons why the authors were motivated to undertake this problem can be found in Chapter I.

The derivation of the mathematical equations which serve both to project a current inventory and to provide the necessary information for using a linear programming technique for computing

least expensive personnel replacements, can be found in Chapters II and III, and Appendix B.

The assumptions considered necessary in the derivation of the mathematical equations can be found in Appendix A.

The procedures utilized in writing a digital computer program for the mathematical equations and the linear programming technique can be found in Chapter IV.

Instructions for the use of the computer program with a detailed description of the necessary inputs and the resultant outputs can be found in Appendix D. A program listing can be found in Appendix E.

The program was run with a representative current inventory with various values of the input parameters, and a sensitivity analysis conducted upon the results. A summary of this analysis can be found in Appendix C. These results were obtained using the U. S. Naval Postgraduate School's CDC 1604 computer.

Conclusions and recommendations can be found in Chapter I, Parts E and F.

The authors wish to express their sincere appreciation to Dr. T. E. Oberbeck, Chairman of the Department of Operations Research, U. S. Naval Postgraduate School, for his particularly valuable technical assistance and guidance in the preparation of this thesis. Acknowledgement is also given to Professor R. Shudde, U. S. Naval Postgraduate School, for his assistance in using the linear programming technique discussed in Chapter IV; to Commander E. D. Napier and Commander J. B. Hansen, Special Projects Office, for their guidance in the formulation of the mathematical model developed in this thesis; and to Captain D. J. Carrison and Captain W. McQuilken, Bureau of Naval Personnel, for their assistance in providing the necessary personnel data and their interest in the possible future applications of this research.

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Column 1 Symbol used in formulation of mathematical model

Column 2 Symbol used in basic computer model

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Column 4 Meaning of symbol

Column 5 Location (page number) in thesis wherein the symbol is discussed

1	2	3	4	5
a_{ij}	-	A(I, J)	Element in the i^{th} row and j^{th} column of a matrix of coefficients.	116
A	-	-	Matrix of coefficients associated with the mathematical technique of linear programming.	37, 116
B_i	-	B(I)	Element in the i^{th} row of a restraint vector.	37, 99
B	-	-	Restraint vector associated with the mathematical technique of linear programming.	37, 117
c_k	-	CC(K)	Cost coefficient associated with the k^{th} pay grade.	78

1	2	3	4	5
c	-	-	Cost vector associated with the mathematical technique of linear programming.	37
k	K	K	Subscript denoting a specific enlisted pay grade.	42
m	M	M	Subscript denoting a specific year of projection.	38
n	N	N	Subscript denoting a specific cell of a (6 x 12) matrix.	29, 44
$P_n(m)$	P(M, N)	P(M, N)	Number of personnel in the n^{th} cell of the projected inventory matrix after m years of projection.	32
$P(m)$	-	-	Projected inventory matrix after m years of projection.	32
$R_j(m)$	R(M, J)	R (M, J)	Personnel requirement used in the formulation of j^{th} element of the restraint vector.	38, 46
$REQ_k(m)$	REQ(M, K)	REQ(M, K)	Personnel requirement for the k^{th} enlisted pay grade in the m^{th} year of projection.	39

1	2	3	4	5
$T_n(m)$	T(M, N)	T(M, N)	Training input to the n^{th} cell of the projected inventory matrix during the m^{th} year of projection.	32
$T(m)$	-	-	Training input matrix for the m^{th} year of projection.	32
X_n	X(N)	X(N)	Personnel in the n^{th} cell of an original inventory matrix	30
X	-	-	Original inventory matrix.	30
α_n	ALFA(N)	ALFA(N)	Attrition operator for personnel in the n^{th} cell of inventory matrix.	29, 49
α	-	-	Attrition operator matrix.	29
β_n	BETA(N)	BETA(N)	Retirement operator for personnel in the n^{th} cell of inventory matrix.	29, 51
β	-	-	Retirement operator matrix.	29
γ_n	GAMMA(N)	GAMMA(N)	Promotion operator for personnel in the n^{th} cell of inventory matrix.	29, 53

1	2	3	4	5
γ	-	-	Promotion operator matrix.	29, 55
δ_n	DELTA(N)	DELTA(N)	Reenlistment operator for personnel in the n^{th} cell of the inventory matrix.	29, 56
δ	-	-	Reenlistment operator matrix.	29, 58
$\tau_k(m)$	TAU(M, K)	TAU(M, K)	Total training input to k^{th} enlisted pay grade during the m^{th} year of projection.	61, 70, 78

CHAPTER I

NON-TECHNICAL DISCUSSION

A. Introduction

1. Definition of the Problem

As the modern Navy becomes more technically oriented, the personnel planner is faced with an increasingly complex problem: how to provide adequate trained enlisted personnel, on time, to meet the requirements of rapidly expanding new weapons systems. While similar problems have always confronted the planner in varying degrees, today's management decisions concerning personnel are more complex than ever before.

The Navy has a serious shortage of qualified manpower for its technical programs. In the past, with the criticality of talent not so acute, the planner could afford some margin for error in his planning. But today he must optimize the utilization of the scarce resources under his control, or soon find a reduced effectiveness of these critical programs due to a lack of qualified personnel.

The technical programs require long lead times for training. This further complicates training planning because input quotas must be determined further in advance, school sttrition rates must be more carefully analyzed, and more accurate accounting of personnel in training must be attained.

The high turnover of personnel in critical skills requires frequent and increasingly costly replacement. A large part of each man's first enlistment in a critical rating is spent in school or some other form of training, and reenlistment rates for these personnel are discouragingly low. To achieve maximum return on this high training investment, there must be a most careful analysis to determine the optimum employment of each man in his particular skill.

Any technical personnel training plan should take into account the future needs of the whole Navy, since the current technical programs may not always have the high priority that they now enjoy. Thus, any personnel planning must remain flexible and carefully consider the future needs of the Navy as they become evident.

For flexibility and reliability in planning, a fast and efficient method of changing the countless variables involved in personnel management is required. The personnel planner, using hand computations alone, is able to consider only one or two alternatives; he then implements the selected plan, and relies on later revisions to make the program suit the Navy's needs. The result is a loss of time, talent and efficiency.

The choice of any training plan should be based not only on its output of trained personnel, but also on its side effects on

such factors as morale, reenlistment rate, cost and utilization of facilities. Only in the optimization of all these factors can the planner employ his personnel resources with maximum efficiency.

The general approach taken in this paper is to construct a mathematical model of a "critical" personnel subsystem, and then to program it on a digital computer. The objective is to provide the personnel planner with a "tool" with which he is able to predict future inventories from a given starting inventory, under known requirements and certain acturial factors. In the process of obtaining the desired projected inventories of the initial group, optimal training inputs are determined.

2. Background

The Operations Analysis Curriculum of the U. S. Naval Postgraduate School requires that each student write a thesis during the second year of study. Two such theses written in 1963 were instrumental in stimulating the authors' interest in the broad area of personnel problems. They were Operations Research and Management of the Navy's Personnel System by LCDR John W. Walden, USN, [1],*and An Analysis of the Cost and Requirements of the Fleet Ballistic Missile Submarine Personnel Subsystem by LT Edwin M. Baldwin, USN, [2].

*Numbers in brackets refer to references in the Bibliography, Page 89.

LCDR Walden's thesis essentially was a broad look at how operations research techniques might be applied to the Navy's complex personnel system, and cited various operations research type studies concerned directly with military personnel management. One of LCDR Walden's objectives was to stimulate further research in the area of applying OR techniques to personnel problems. Among his recommendations, he stated:

> a necessary first step has to do with studying the (personnel) system in its entirety. Some model of our personnel system must be developed and some meaningful measures of effectiveness derived.

The authors were further motivated to investigate the Navy's personnel problems by their summer field trips to Washington, D. C., during the period 1 June to 12 July 1963, as part of the Operations Analysis Curriculum. LCDR's Meeks and Metcalf were assigned to the Bureau of Naval Personnel, while LT's Greenhalgh and Harper were attached to the Special Projects Office. In the course of the summer assignments, the officers assigned to BuPers became aware of the compelling need for the Navy's personnel planners to be able to project personnel inventories under the constraints of varying policy changes and special programs. The officers assigned to the Special Projects Office gained an appreciation for the increasingly acute personnel problems associated with the rapidly expanding demand for the critically short,

highly qualified, highly trained personnel of our FBM program.

While in Washington, the authors attended a briefing at the Special Projects Office where a paper entitled Nuclear Officer Training Requirements Program by CDR Charles E. Woods, USN, [3] was presented. Briefly, this paper was concerned with the optimization of the numbers of nuclear-trained officers in various seniority groups to meet the demands of the expanding SSBN and SSN building programs.

This study was of paramount importance in the recent reevaluation of the training plans for officers entering the SSBN and SSN programs. CDR Woods, a graduate of the Operations Analysis Curriculum of the U. S. Naval Postgraduate School, utilized many operations analysis techniques in his study. His paper was therefore of interest because it represented an example of how these techniques could be successfully applied to the officer personnel system. It was also of interest for the possible application of a similar model to the enlisted personnel system.

Out of a recognition of the vital importance of the personnel subsystem to any weapons system, a desire to apply operations research techniques to the Navy's personnel system, and a mutual interest in "people", the four authors were motivated to undertake this paper.

B. Concept of PIPE (Personnel Inventory Projection, Enlisted)

1. Objective of the Model

The Navy personnel planner needs a method which will quickly, accurately and with broad flexibility:

a. Project inventories of personnel for a desired time period;

b. Compute optimal training inputs under known requirements and current acturial factors; and,

c. Predict excesses and shortages of the projected inventory by pay grade.

The objectives of PIPE are to accomplish the above with a digital computer program which will require a minimum of running time, be simple to operate and modify according to the user's requirements, and employ imputs in a form readily available to the user. In other words, it is hoped that, by using this model, the personnel planner can get a rapid and reasonable estimate of the personnel situation of the future, and experiment with ways of improving this situation as he sees fit.

2. Classification of Naval Enlisted Personnel

The structure of the Naval enlisted personnel system is complex and varied, with great differences between men as to experience level, skills, and criticality. In order to predict the number of men in this structure at some future date, it is first

necessary to examine the personnel system for some kind of consistency of qualities so that the men can be grouped and examined with respect to recurring phenomena. A breakdown by ratings alone may not be sufficient for this purpose, because some ratings may include a broad spectrum of specific skills, some of which may be in critical supply, and some of which may not.

Within the critical ratings, however, the Naval Enlisted Classification Code (NEC) does provide such a grouping of personnel. Men with a common NEC share comparable skills and criticality; they also share approximately the same attrition, reenlistment, retirement, and promotion factors. Therefore, the enlisted men considered in this model may be delineated by NEC in the critical ratings, or by any other criteria that provides similar characteristics.

3. The Inventory Matrix

The men in a specific group (i. e., with common acturial factors) are next further subdivided by both pay grade and years of obligated active service. This delineation lends itself to a matrix formulation of the initial inventory. The matrix consists of cells representing all personnel in the group, with the numbers in appropriate cells representing personnel according to their present obligated service and pay grade. Within each pay grade, the matrix is further categorized by specified enlistments. This allows

personnel in the inventory to be delineated by total years of active service as well as by pay grade and obligated service. In the model, total years of service are analogous to current enlistment, where it is assumed that all enlistments are of six years duration.

Hereafter, the word "inventory" will always denote this type of matrix description of the particular group of personnel under consideration, with cells of the matrix representing numbers of personnel according to their obligated service, pay grade, and current enlistment. This type of formulation is considered necessary because this model deals with people, and as such, a means must be provided for accounting for the differences between men within the group. It is felt that such a matrix expresses the differences in professional capabilities of the personnel in the group.

4. Change of Inventory With Time

As indicated above, a given cell of the inventory matrix represents the number of personnel categorized by:

a. Obligated service

b. Pay grade

c. Current enlistment

With the passage of an increment of time there will be a change in the inventory matrix because at least one of the categories above will change for all personnel. Thus, a man who is represented in the inventory of a given cell at a certain time will necessarily be

represented in a different cell after an increment of time. This change of the inventory with time, then, can be visualized as a "movement" of personnel from one cell to another within the inventory matrix. Throughout the remainder of this paper, the "movement" of personnel within the inventory matrix will refer to this concept of the change of the inventory with time.

Because personnel in the critical ratings require approximately one year of formal schooling or other training before they are ready to join the operating forces, it was decided to look at time in discrete increments of one year.

There are four basic acturial factors or rates, which are a characteristic of any particular inventory in the personnel system. They are defined as follows:

a. Attrition rate: a percentage of men who leave the inventory due to death, sickness, disciplinary action, lack of aptitude, etc.

b. Retirement rate: the percentage of men who leave the service by retiring after 20 years or more of active service.

c. Promotion rate: the percentage of men who are promoted from any one pay grade, to the next higher pay grade.

d. Reenlistment rate: the percentage of men who reenlist in the Navy for a period of six years and thus remain in the inventory.

It is the application of these actuarial factors in the appropriate manner which causes the change in the basic inventory per unit time. Thus, it can be said that these four rates cause personnel to "move" through the inventory matrix. In the sense that these factors cause a change in the inventory matrix with time, they can be thought of as "operating" on the basic inventory matrix to "transform" it into a new, or changed inventory. Accordingly, these basic actuarial rates of attrition, reenlistment, retirement, and promotion will be referred to as "operators".

Upon examining these four operators as they affect the inventory matrix, the following conclusions result:

a. The attrition operator will affect every cell in the inventory matrix, serving to reduce the number of people in each cell as time passes.

b. The retirement operator will affect only those cells which represent personnel with 20 years of active service or more.

c. The promotion operator will apply to all cells which are determined to contain men eligible for promotion.

d. The reenlistment operator will apply only to those cells which represent men in their last year of obligated service.

It is readily seen that the "operators" must be applied to different cells in different ways in order to cause movement through the inventory matrix in a realistic manner. Thus each

operator takes on the form of the inventory matrix, whereby each cell of the inventory matrix has a corresponding attrition, retirement, promotion, and reenlistment rate associated with the respective operator matrices. Through a proper means of indexing, the appropriate operators are thereby applied to their corresponding inventory cells, and the personnel represented in the inventory matrix will be "moved" through the matrix with the passage of time.

Due to the action of the operators, the numbers in all cells of the inventory matrix will be reduced to reflect attrition, and the numbers in certain cells will be reduced to reflect retirement and non-reenlistment. To replace these men there must be a means for inserting new men into the program in any given year. This is done with training inputs, which again take the form of a matrix corresponding to the inventory matrix. In this way numbers in selected cells of the inventory matrix can be increased and these cells can be varied to reflect the user's choice of training inputs.

5. The Projection Equations

The passage of time:

a. Generates movement of personnel through the cells of the inventory matrix under the influence of the operator matrices;

b. Provides for the addition of personnel as training inputs.

With this knowledge, projection equations can be derived that express the projected inventory in terms of the original inventory, operators, and training inputs. The only unknowns of these equations are the training inputs.

In order to utilize these equations, personnel requirements for the future must be made available. Basically, these requirements emanate from the Department of Defense in The Five-Year Force Structure and Financial Program. This model uses these requirements grouped by pay grades for each year of projection.

If the requirements are set as linear bounds on the projection equations, then by a mathematical technique known as linear programming, the unknown training inputs can be determined in such a way as to satisfy the requirement in a manner which is optimal with respect to cost and/or numbers of personnel.

With the solutions to the optimal training inputs available, the projection equations then can be used to express explicitly the projected inventory.

C. Other Personnel Computer Models

Three separate computer models generally concerned with the projection of personnel were studied prior to the conception of the models developed in this thesis. A brief description of these systems is considered appropriate in order to place the subject model in perspective.

1. Project MOON

Project MOON (Meeting of Operational Needs) is an enlisted personnel simulation system that was developed by the American Institute of Research under contract from the Personnel Research Division (Pers 15) of the Bureau of Naval Personnel. MOON provides for the projection of a selected rating for any period of from one to five years into the future. These projections are made either (1) on the basis of the continuance of current policies, planning methods, and actuarial factors, or (2) with the introduction of selected changes in any of these policies, planning methods, or actuarial factors. Thus, MOON can be used to estimate the future state of a selected rating under current conditions, or test the effects of the changes in these conditions over the desired period of projection.

One of the essential features of the MOON program is that it operates on "real" people; the basic starting sample population consists of extracted service records taken from the BuPers

magnetic tape master file. Each "person" in the sample population is denoted by a sequence of alpha-numeric characters that denote nine basic simulated personnel variables which are:

a. Pay grade

b. Primary Naval Enlisted Classification Code (NEC)

c. Secondary NEC

d. ETST Score (or MECH)

e. USN/USNR

f. Time in grade

g. Obligated active service

h. Length of service

i. Sum of the basic test battery scores, (GCT, ARI, and ETST or MECH)

Other inputs to the program consist of fourteen directly controllable variables which represent the various policy and planning methods previously mentioned, and seven indirectly controlled variables which represent the actuarial factors involved.

The main program of Project MOON simulates the passage of time by performing various peripheral routines on the appropriate sample population. The actual people, (i.e., extracted records), considered by the main simulation program are selected randomly by means of Monte Carlo techniques from the population sample.

The basic input of each projection run is a set of tabulations representing the state of selected ratings at the start and at the end of each fiscal year of the projection period. These reports are cross-tabulated in various ways, producing an awesome array of data, including various histograms.

In summary, Project MOON is a highly sophisticated, complex program which is considered to have two particular limitations:

a. The running rime (computer time) per year of projection for one selected rating is about one hour; hence, a five-year projection requires about five hours of computer time. This is a function of the complexity of the program, particularly with regard to the large number of random selections made in the program.

b. Both the annual updating procedure and the annual validation procedure for MOON require massive manual data-handling efforts.

2. The Nuclear Officer Training Requirements Program

The Nuclear Officer Training Requirements Program [3] is the computer model which was developed from the mathematical model conceived by CDR Charles E. Woods, USN, who was then attached to the Office of the Chief of Naval Operations (Op-31).

Commander Wood's model was programmed by Booz-Allen Applied Research,Inc., Washington, D. C., under contract from the Special Projects Office.

This model uses elementary matrix methods to predict, year by year, the number of nuclear trained officers in the various seniority groups, based on assumed inputs into the training program and estimates of attrition for the different seniority groups. By varying the input parameters, the effects on the distribution of officers in various seniority groups can be examined. The predicted numbers by seniority groups are then compared with known requirements. Estimated requirements for trained officers are based on known shipbuilding schedules and established or postulated seniority criteria. The inputs of officers entering training for successive years are treated as unknowns.

This model uses linear programming techniques to minimize the total number of officers requiring nuclear training based on the requirements, seniority criteria and attrition factors. The outputs of the computer model is training requirements by seniority groups per fiscal year.

3. <u>SMS Personnel Projection Model</u>

The Surface Missile Systems Personnel Projection Model is an elementary, deterministic computer model which was developed for the Special Surface Missile Systems Task Forces (Pers 17) by the computer facility at the Naval Weapons Laboratory,

Dahlgren, Virginia. Although the authors were able to obtain a program listing for this model, the model is undocumented as far as basic explanation and concept are concerned.

Prior to the development of the computer model, Pers 17 undertook the major task of purifying its basic inventory of trained personnel by attempting to identify adequately all personnel who had ever been trained for one of the critical NEC's involved in the surface missile systems. With their inventory purification complete, they next developed a simplified model which would project their inventory into the future under certain known acturial rates such as attrition, reenlistment and retirement. The program's output consists of predicted totals of personnel in various NEC's for each six-month increment into the future.

D. Comparison of Models

PIPE is considerably less sophisticated than Project MOON; it does not consider as many variables, and it does not attempt to project the inventory of an entire rating. It does, however, consider the important factors in the projection of personnel inventories, and because of its simplicity it requires only a few minutes to run on the computer. Also, it does not require the massive accumulation of data that MOON requires.

The SMS projection model of Pers 17 treats an NEC as a single entity, but does not consider requirements or training inputs.

PIPE is more sophisticated in that the NEC's are delineated by pay grade and are compared to requirements to obtain optimal training inputs for every year of projection.

The Nuclear Officer Training Program model was designed to project inventories of officers; it cannot be directly transferred for use as a projection of enlisted men due to the added complexity of the enlisted personnel system. But just as CDR Wood's model projected inventories of officers by submarine billets over a period of time, so PIPE will project inventories of enlisted personnel by pay grades in NEC's or similar groups. Thus, there is a similarity between the two models in concept and approach, but PIPE is considered more complex due to the greater complexity of the enlisted personnel system.

E. Conclusions

1. Linear programming is a technique traditionally associated with the optimal allocation of resources with respect to minimum cost and/or other requirements. PIPE demonstrates that linear programming can provide valuable assistance to the personnel planner concerned with the projection of current inventories into the future, the necessary personnel inputs over the years of projection, and the problem of ensuring that the projected inventories meet the requirements of the future. PIPE specifically provides estimates of the annual training inputs necessary to meet the future requirements of critical rating/NEC groups at minimum cost, where costs are measured either in number of personnel or monetary units.

2. PIPE is a unique program for the projection of personnel inventories because it considers requirements five years in advance of the present time in deriving its minimum cost solution. Therefore, PIPE is a long-range planning tool in that it "looks ahead" to the future and computes the best solution to the problem considering the requirements for personnel of each year for five years.

3. The most difficult part of any personnel inventory projection of an enlisted personnel subsystem is the initial breakdown and categorizing of the personnel in the subsystem. While generalization leads to inaccuracies, it is also true that great detail brings

about intricately complicated and time-consuming computer programs. PIPE is sufficiently detailed to be useful to the personnel planner, yet basic enough to require simple inputs and a minimum of computer time.

4. It is recognized that many complex prediction equations have been developed in this thesis, and that no matter how logically they have been derived, they can not be considered dependable until their solutions are exposed to the realities of empirical data. PIPE was operated with actual input data from BuPers for Fiscal Year 1963; it cannot be validated until historical data becomes available.

5. The use of obligated service as a criterion for the accounting of personnel in the inventory allows an accurate and efficient use of reenlistment rates as input parameters in the projection of personnel inventories. No matter what the length of a man's time in service, or his pay grade, or his enlistment, he must either reenlist or leave the service during the last year of his enlistment. The obligated service criterion pinpoints him in the inventory in such a manner that the reenlistment rate can be directly applied to him when he is in his last year of obligated service. PIPE reflects the use of this criterion.

6. The obligated service criterion allows training input to be delineated further than has heretofore been the case. Training inputs to the subsystem can therefore be specified by their length of service, pay grade, and obligated service, thereby allowing more specific control over these inputs to a personnel projection.

7. Experience has shown the personnel planner that controlled variations of promotion, retirement, reenlistment, and attrition rates cause predictable trends in the training inputs and associated costs necessary to meet future personnel requirements. Using the actual current inventory of a critical rating, with various values of the input parameters, (promotion, retirement, reenlistment and attrition rates), the resultant outputs of the model were analyzed with respect to the sensitivity of the solutions to the actual values of the input parameters. The details of this sensitivity analysis are described in Appendix C. This analysis supports the aforementioned trends, and hence the model is considered to use these rates effectively in the projection of an inventory; it shows that the model reflects for all cases considered the proper direction of the trends anticipated for the various input parameters.

F. Recommendations

1. Test the validity of the model using historical data. When the information becomes available for personnel inventories separated by at least one year, then the model should be tested for its approximation of the real life situation.

2. Investigate the possibility of computing reenlistment, retirement, and attrition rates from information on the BuPers master magnetic tape files.

3. Investigate the actual distribution of training input by pay grade, obligated service, and length of service, in order to evaluate the validity of the uniform-distribution assumption used in the model.

4. Operate the model with various training-input distributions, and analyze their effects on the solution with respect to the total training input, and excesses and shortages in the different pay grades.

5. Investigate the various methods by which the relative costs of the training inputs from the different pay grades can be computed. Since these costs serve to control the linear programming solution, the accuracy with which these costs are determined will directly affect the utility of the model.

6. The linear program in this model uses restraint equations with many lower limits, while the only upper limit represents the permissible excess of inventory over requirements. Investigate the feasibility of placing additional upper limits on the restraint equations. These upper limits could correspond to the influences of such factors as the capacity of training facilities, and the availability of personnel in certain pay grades.

7. With a very minor program change, which would enable perfectly arbitrary training plans to be an input, project current inventories with a proposed training plan and compare the projected inventories with requirements. This procedure enables the personnel planner to compare numerous training plan proposals.

CHAPTER II

BASIC MODEL

A. Concept

The basic problem being undertaken is one of complex inventory control, characterized by the following facts:

1. Items in the inventory are people.
2. People with different professional capabilities must be subdivided to provide distinction.
3. The number of people to be found in the inventory is affected by numerous factors, the values of which depend on the particular people under consideration.
4. During the passage of time, the people assume new potentials, and this change can occur in various ways.

The initial problem is that of creating a method of tabulating the inventory in a manner which is compatible with the characteristics enumerated above. The method utilized by the Officer Model discussed in Chapter I uses "years of completed service" to subdivide the inventory as to professional capabilities. This characterization is inadequate for enlisted men. The criterion for selection of subdivision rules must be such as to provide the user of the model with the maximum usable information concerning the professional potential of the inventory.

The term professional potential as it applies to the enlisted personnel inventory requires definition. The various billets required to be filled by a given inventory vary in complexity and demand different levels of education and experience. Additionally it is important that an inventory specify the expected tenure of service of each man. Professional potential is therefore defined as a function of educational level, experience level, and expected tenure of service. It is obvious that the

single parameter of "years of completed service" as used in the Officer Model is inadequate to define sufficiently the professional potential of an enlisted man.

Cdr. E. Napier, of the Navy's Special Projects Office, suggested subdividing the inventory by enlisted pay grade and years of obligated service. Such a representation is shown in Figure II-1. This subdivision, although satisfactory at first glance, does not provide for sufficient breakdown by experience level. This can best be illustrated with an example.

Years of Obligated Service	E-3	E-4	E-5	E-6	E-7
1					
2					
3					
4					
5					
6					

Figure II-1
Subdivision of Inventory

Consider 100 men of pay grade E-6 with three years obligated service. It would be far more meaningful to state that of these 100 men there are 15 men with about two to four years service, 60 men with about eight to ten years service, and 25 men with about 14 to 16 years service. This information provides a much more informative description of the experience level of the inventory and allows the personnel administrator to appreciate more fully the professional potential of the

total number of men in any one pay grade.

This then leads to the subdivision of the inventory matrix as indicated in Figure II-2, wherein the columns within any one pay grade are designated as successive enlistments. It will be noted that the number of columns, and hence enlistments, which are associated with each pay grade varies. By analyzing representative current inventories of several critical ratings through information provided by the Bureau of Naval Personnel, it was determined that personnel on certain enlistments are not expected to be found in certain pay grades. This information dictated the specific matrix as shown in Figure II-2.

In order to use this inventory array it is necessary to create some notational scheme to represent each cell in the array. If this problem is approached by treating the array as a conventional matrix and using double subscripts to identify each cell within the array, the result proves to be overwhelming in notational complexity. This leads then to the scheme of assigning a single subscript to each cell in the order indicated in Figure II-3, which simplifies the notational difficulties immeasurably, and more importantly, provides greater flexibility in the ultimate writing of a digital computer program for the mathematical model. This array is described as a 6x12 matrix with 72 cells.

With the passage of time, the number of people to be found in any cell will change providing there is no additional input. The reasons for this change can be attributed to four factors:

1. attrition rate
2. retirement rate
3. promotion rate
4. reenlistment rate

In the model, these factors will be represented by so-called matrix operators. The subdivision of the inventory array by cells demands that a separate set of

Years Obligated Service	Pay Grade E-3	Pay Grade E-4		Pay Grade E-5		Pay Grade E-6			Pay Grade E-7			
	First Enlist-ment	First Enlist-ment	Second Enlist-ment	First Enlist-ment	Second Enlist-ment	First Enlist-ment	Second Enlist-ment	Third Enlist-ment	Second Enlist-ment	Third Enlist-ment	Fourth Enlist-ment	Fifth Enlist-ment
1												
2												
3												
4												
5												
6												

Figure II-2

Inventory Matrix

Years Obligated Service	Pay Grade E-3	Pay Grade E-4		Pay Grade E-5		Pay Grade E-6			Pay Grade E-7			
	First Enlist-ment	First Enlist-ment	Second Enlist-ment	First Enlist-ment	Second Enlist-ment	First Enlist-ment	Second Enlist-ment	Third Enlist-ment	Second Enlist-ment	Third Enlist-ment	Fourth Enlist-ment	Fifth Enlist-ment
1	6	12	18	24	30	36	42	48	54	60	66	72
2	5	11	17	23	29	35	41	47	53	59	65	71
3	4	10	16	22	28	34	40	46	52	58	64	70
4	3	9	15	21	27	33	39	45	51	57	63	69
5	2	8	14	20	26	32	38	44	50	56	62	68
6	1	7	13	19	25	31	37	43	49	55	61	67

Figure II-3

Subscripted Inventory Matrix

values for these operators be provided for each cell. The following definitions are therefore necessary:

$n \equiv$ the number of the cell in the matrix under consideration ($n \equiv 1, 2, \ldots, 72$)

$\alpha_n \equiv$ attrition rate for cell number n

$\alpha \equiv [\alpha_n] \equiv$ complete 6x12 array of attrition rates

$\beta_n \equiv$ retirement rate for cell number n

$\beta \equiv [\beta_n] \equiv$ complete 6x12 array of retirement rates

$\gamma_n \equiv$ promotion rate for cell number n

$\gamma \equiv [\gamma_n] \equiv$ complete 6x12 array of promotion rates

$\delta_n \equiv$ reenlistment rate for cell number n

$\delta \equiv [\delta_n] \equiv$ complete 6x12 array of reenlistment rates

By these definitions the necessary specification of any operator demands an array similar to the inventory matrix. For example

$$\alpha = [\alpha_n] = \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline
\alpha_6 & \alpha_{12} & \alpha_{18} & \alpha_{24} & \alpha_{30} & \alpha_{36} & \alpha_{42} & \alpha_{48} & \alpha_{54} & \alpha_{60} & \alpha_{66} & \alpha_{72} \\ \hline
\alpha_5 & \alpha_{11} & \alpha_{17} & \alpha_{23} & \alpha_{29} & \alpha_{35} & \alpha_{41} & \alpha_{47} & \alpha_{53} & \alpha_{59} & \alpha_{65} & \alpha_{71} \\ \hline
\alpha_4 & \alpha_{10} & \alpha_{16} & \alpha_{22} & \alpha_{28} & \alpha_{34} & \alpha_{40} & \alpha_{46} & \alpha_{52} & \alpha_{58} & \alpha_{64} & \alpha_{70} \\ \hline
\alpha_3 & \alpha_9 & \alpha_{15} & \alpha_{21} & \alpha_{27} & \alpha_{33} & \alpha_{39} & \alpha_{45} & \alpha_{51} & \alpha_{57} & \alpha_{63} & \alpha_{69} \\ \hline
\alpha_2 & \alpha_8 & \alpha_{14} & \alpha_{20} & \alpha_{26} & \alpha_{32} & \alpha_{38} & \alpha_{44} & \alpha_{50} & \alpha_{56} & \alpha_{62} & \alpha_{68} \\ \hline
\alpha_1 & \alpha_7 & \alpha_{13} & \alpha_{19} & \alpha_{25} & \alpha_{31} & \alpha_{37} & \alpha_{43} & \alpha_{49} & \alpha_{55} & \alpha_{61} & \alpha_{67} \\ \hline
\end{array}$$

With these definitions established it is possible to consider the ensuing movement of people from one cell to another during the passage of time. In order to determine how to represent the change in inventory during a passage of time it is helpful to specify the conditions under which a change in inventory can occur. These

conditions are threefold and are specifically:

1. The passage of one year permits a man to move to the next cell in the array, subject to the restrictions of attrition, retirement, and reenlistment;
2. The movement described in (1) plus an additional move into the next higher pay grade by promotion;
3. The addition to a cell from outside the array as a result of a training input.

The nature of the inventory array dictates clearly which of conditions (1) or (2) can prevail for each and every cell. These are illustrated in Figure II-4. There exist two basically different flows, the existence of which can be appreciated from Figure II-4 as one notes that the line "due to promotion" is discontinuous; the first part of which is associated with cells n=8 to 13, the second part of which is associated with cells n=20 to 61. Specifically this discontinuity is due to the varying number of enlistments to be found in any one pay grade, and hence the varying number of cells to be included in a jump from any one pay grade to the next. The cells for which there is no corresponding point on the line "due to promotion" are therefore cells to which one cannot be promoted, an example of which is n=16. The existence of these cells can be appreciated by observing exactly what enlistments the cells are part of and then the non existence of the enlistments from which their inputs would have had to come. There also exist in Figure II-4, cells for which there can be no input except a training input, an example being n=7. Every cell can receive a training input, condition (3), and is therefore not included in Figure II-4.

These concepts necessitate the following definitions:

$X \equiv$ original inventory matrix (6x12)

$X_n \equiv$ original inventory in cell number n (n=1, 2, ..., 72)

$\therefore X \equiv [X_n]$

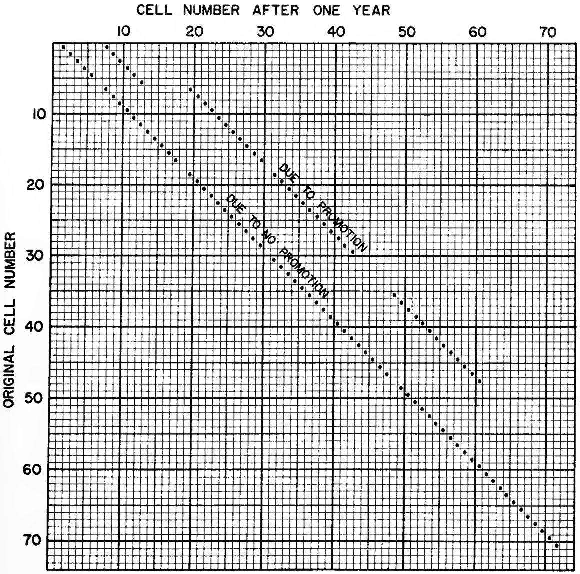


FIGURE II-4 ONE YEAR

MOVEMENT GRAPH

$P(m) \equiv$ projected inventory matrix (6x12) at the end of the m^{th} year of projection

$P_n(m) \equiv$ projected inventory in cell number n at the end of the m^{th} year of projection (n= 1, 2, ..., 72)

$\therefore\ P(m) \equiv [P_n(m)]$

$T(m) \equiv$ training input matrix (6x12) during the m^{th} year of projection

$T_n(m) \equiv$ training input into cell number n during the m^{th} year of projection (n= 1, 2, ..., 72)

$\therefore\ T(m) \equiv [T_n(m)]$

The equations which will be written in terms of these quantities will ultimately be programmed for a digital computer. Since these programs must be written in capital letters, the symbols used above to define the cells of the matrices are capital letters vice the more common usage of lower cased letters.

With these definitions established the next step is to construct the mathematical equations that will describe the projected inventory matrix, cell by cell ($P_n(m)$), as a function of the cells of the original inventory matrix (X_n), the cells of the operators (α_n, β_n, γ_n, δ_n), and the training input for each of the m years of projection ($T_n(m)$). The initial time period to be considered is one year (m= 1).

When these equations have been derived, a comparison will then be possible between the projected inventory and the stated requirements. If the training inputs for each cell ($T_n(1)$) are unknowns, then the projection equations, when compared with the requirements, yield the necessary inputs to a linear program, a mathematical technique that will minimize the training inputs that are necessary such that the projected inventory ($P(1)$) meets the requirements. Having found the solution for the only unknowns in the projection equations, (training inputs), the projected inventory can then be expressed explicitly.

B. Formulation

The equation representing the inventory of each cell in the projected inventory matrix depends on which cell is being considered. Therefore, it is helpful to repre-

sent the components of the equation for each cell with a flow chart. Figure II-5 shows the four possible inputs to any cell, $P_n(1)$, of the projected inventory after one year of projection. For a given cell, $P_n(1)$, only a subset of these inputs are applicable. For this reason each subsection of Figure II-5 is explained in detail:

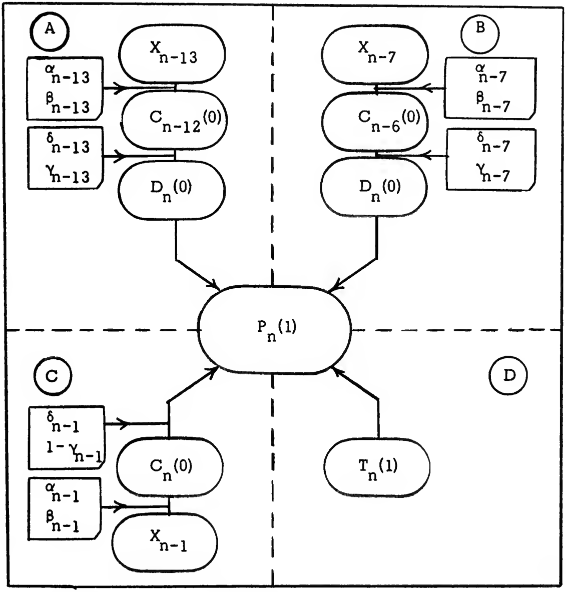


Figure II - 5

Projection Flow Chart

Figure II-5A

If promotion is possible from cell X_{n-13} of the original inventory, then the inventory in cell X_{n-13} must be subjected to attrition and retirement rates. That part of X_{n-13} that survives these rates is defined as $C_{n-12}(0)$.

$$\therefore \quad C_{n-12}(0) = \left[1 - (\alpha_{n-13} + \beta_{n-13})\right] X_{n-13}$$

$C_{n-12}(0)$ represents that part of X_{n-13} that is eligible for promotion. Applying promotion and reenlistment rates to $C_{n-12}(0)$, that part that is promoted and reenlisted is defined as $D_n(0)$.

$$\therefore \quad D_n(0) = \delta_{n-13}\, \gamma_{n-13}\, C_{n-12}(0)$$

$D_n(0)$ is a part of $P_n(1)$ if this move is possible.

Figure II-5B

If promotion is possible from cell X_{n-7} of the original inventory, then the inventory in cell X_{n-7} must be subjected to attrition and retirement rates. That part of X_{n-7} that survives these rates is defined as $C_{n-6}(0)$.

$$\therefore \quad C_{n-6}(0) = \left[1 - (\alpha_{n-7} + \beta_{n-7})\right] X_{n-7}$$

$C_{n-6}(0)$ represents that part of X_{n-7} that is eligible for promotion. Applying promotion and reenlistment rates to $C_{n-6}(0)$, that part that is promoted and reenlisted is defined as $D_n(0)$.

$$\therefore \quad D_n(0) = \delta_{n-7}\, \gamma_{n-7}\, C_{n-6}(0)$$

$D_n(0)$ is a part of $P_n(1)$ if this move is possible.

Figure II-5C

If the passage of one year can move the inventory in cell X_{n-1} into $P_n(1)$, then X_{n-1} must be subjected to attrition and retirement rates. That part of X_{n-1} which survives these rates is defined as $C_n(0)$.

$$\therefore \quad C_n(0) = \left[1 - (\alpha_{n-1} + \beta_{n-1}) \right] X_{n-1}$$

$C_n(0)$ represents that part of X_{n-1} that is eligible for promotion. Applying promotion rates and reenlistment rates to $C_n(0)$, that part that is reenlisted but <u>not</u> promoted is a part of $P_n(1)$ if this move is possible.

Figure II-5D

All cells, $P_n(1)$, are eligible for a training input, $T_n(1)$.

By using Figure II-4 to determine which subset of the possible inputs shown in Figure II-5 is applicable to each cell $P_n(1)$, $(n = 1, 2, \ldots, 72)$, the following set of projection equations are derived:

$$P_n(1) = \begin{cases} \delta_{n-1} \left[1 - \gamma_{n-1} \right] C_n(0) + D_n(0) + T_n(1) & n \neq 1,7,19,31,49 \\ D_n(0) + T_n(1) & n = 49 \\ T_n(1) & n = 1,7,19,31 \end{cases}$$

where

$$C_n(0) = \begin{cases} \left[1 - (\alpha_{n-1} + \beta_{n-1}) \right] X_{n-1} & n \neq 1,7,19,31,49 \\ 0 & n = 1,7,19,31,49 \end{cases}$$

$$D_n(0) = \begin{cases} \delta_{n-7} \gamma_{n-7} C_{n-6}(0) & n = 8 - 13 \\ \delta_{n-13} \gamma_{n-13} C_{n-12}(0) & n = 20\text{-}30, 32\text{-}43, 49, 61 \\ 0 & n = 1\text{-}7, 14\text{-}19, 31, 44\text{-}48, 62\text{-}72 \end{cases}$$

With the projection equations defined for every cell of the projected inventory matrix, the next step is to compare these equations against the stated requirements, allowing the training inputs to be unknowns. This provides the necessary inputs to a linear program. The linear programming method that is utilized

is as defined in GASS, [4]. In matrix formulation this method is used to

$$\text{minimize } cY \quad , \quad \text{subject to} \quad AY \geq B$$

where

$c = [c_j] \equiv$ Cost Coefficient Vector

$Y = [Y_j] \equiv$ Unknown

$cY = [c_j\, Y_j] \equiv$ Objective Function

$A = [a_{ij}] \equiv$ Matrix of Coefficients

$B = [B_i] \equiv$ Restraint Vector

and in this problem

$AY \geq B$ represents an array of equations generated from comparing the projection equations with the stated requirements, with the Y matrix representing the unknown training input.

The necessary inputs to the linear program are therefore

the matrices A, B, c, and cY. It is therefore these matrices that must be derived from the projection equations and the requirements.

The method of linear programming is dependent on the manner in which the requirements are stated. The problem can be approached assuming that the requirements could be stated for each of the 72 cells of the projected inventory matrix. The resultant matrices A, B, c, c Y are discussed in Appendix II; however, this approach is not pursued further as this form of stating requirements is not anticipated to be realistic. This leads to a form wherein the requirements are stated in terms of requirements for entire pay grades. These requirements are defined as

$R_1(m)$ = The sum of the total requirements for each of the pay grades E-3 through E-9 for the m^{th} year of projection.

$R_2(m)$ = The sum of the total requirements for each of the pay grades E-4 through E-9 for the m^{th} year of projection.

$R_3(m)$ = The sum of the total requirements for each of the pay grades E-5 through E-9 for the m^{th} year of projection

$R_4(m)$ = The sum of the total requirements for each of the pay grades E-6 through E-9 for the m^{th} year of projection.

$R_5(m)$ = The sum of the total requirements for each of the pay grades E-7 through E-9 for the m^{th} year of projection.

It should be noted that this form of stating requirements demands that an input to the model must be the total requirement for each of the pay grades. The following definitions are therefore in order:

$REQ_1(m)$ = Total requirement for pay grade E-3 for the m^{th} year of projection.

$REQ_2(m)$ = Total requirement for pay grade E-4 for the m^{th} year of projection.

$REQ_3(m)$ = Total requirement for pay grade E-5 for the m^{th} year of projection.

$REQ_4(m)$ = Total requirement for pay grade E-6 for the m^{th} year of projection.

$REQ_5(m)$ = Total requirement for pay grade E-7 through E-9 for the m^{th} year of projection.

and therefore

$$R_1(m) = \sum_{k=1}^{5} REQ_k(m)$$

$$R_2(m) = \sum_{k=2}^{5} REQ_k(m)$$

$$R_3(m) = \sum_{k=3}^{5} REQ_k(m)$$

$$R_4(m) = \sum_{k=4}^{5} REQ_k(m)$$

$$R_5(m) = REO_5(m)$$

This procedure for the specification of requirements appears to be the most useful as it approaches the problem from an operational point of view. Merely stating the requirements for individual pay grades offers the following dilemma.

Consider the requirements for pay grade E-5 to be 1000 men and for pay grade E-6 to be 500 men, and the projected inventory to be 800 men in the E-5 and 700 men in E-6. Such a situation would fail to meet the requirements for pay grade E-5; however, in actuality, since there exists an excess of 200 men in E-6, the requirements are really oversatisfied with E-6 personnel filling E-5 billets. By pyramiding the requirements, as defined by the $R_k(m)$ above, this dilemma is eliminated. Such a form of stating requirements will, however, necessitate consideration as to the varying costs of training personnel in different pay grades. A discussion of this cost analysis can be found in Appendix A, from which is generated the definition:

c_k = Cost coefficient associated with the k^{th} pay grade $(k = 1, 2, \ldots, 5)$

$c = \left[c_k \right]$

where $k_1 \rightarrow E - 3$

$k_2 \rightarrow E - 4$

$k_3 \rightarrow E - 5$

$k_4 \rightarrow E - 6$

$k_5 \rightarrow E - 7$ through $E - 9$

With a specific form of requirements available, with associated cost coefficients, it is possible to derive the matrices A, B, c, cY, and hence the necessary inputs to a linear program which will compute the minimum training input necessary in order to meet the requirements. This derivation, with a listing of the appropriate solutions is shown in detail in Appendix B.

With the training inputs now available, the projection equations will determine the projected inventory at the end of one year of projection.

C. Inputs and Outputs

In this section the inputs and outputs of the basic model are delineated and defined.

1. Inputs

a. Initial Inventory

The initial inventory is the basic group of personnel under consideration whose future composition is of interest to the personnel planner. This group is subdivided by pay grade, obligated service, and current enlistment into a matrix of 72 cells as shown in Figure II-3. In this thesis the term "initial inventory" can apply to any logical subdivision of the personnel system that can be described as "critical", in the same sense that any scarce but valuable commodity might be regarded as "critical". Thus the initial inventory may be composed of:

(1) A critical rating (i. e., the ET rating);

(2) A critical NEC (i. e., NEC 3322, Inertial Navigation Subsystem Technician, Mk II);

(3) Any desired combination of ratings and/or NEC's.

The criteria for allocating individuals of the subject personnel group into their appropriate rows and columns (i. e., cells) of the inventory matrix are given below:

(1) Row Criterion

This is a breakdown by obligated service. An individual in the inventory is represented by a row numbered $(n+1)$ according to the relationship:

$$n = (\text{calendar year of expiration of active obligated service}) - (\text{present calendar year})$$

where n can take on the values $n = 0, 1, 2, 3, 4, 5$.

Example: A person in a calendar year 1964 inventory whose expiration of active service (EAOS) occurs in calendar year 1967 would be placed in row number four. That is,

$$n = 1967 - 1964 = 3, \text{ so that}$$

$$\text{row } (n+1) = \text{row } (4).$$

(2) Column Criterion

This is a breakdown by pay grade and current enlistment.

COLUMN	PAY GRADE	CURRENT ENLISTMENT	LENGTH OF SERVICE X(IN YEARS)
1	E-3*	First	$0 < X \leq 6$
2	E-4	First	$0 < X < 6$
3	E-4	Second	$X \geq 6$
4	E-5	First	$0 < X < 6$
5	E-5	Second	$X \geq 6$
6	E-6	First	$0 < X < 6$
7	E-6	Second	$6 \leq X < 12$
8	E-6	Third	$X \geq 12$
9	E-7**	Second	$6 \leq X < 12$
10	E-7**	Third	$12 \leq X < 18$
11	E-7**	Fourth	$18 \leq X < 24$
12	E-7**	Fifth	$24 \leq X < 30$

Notes: * Pertains to pay grades E-1, 2 and 3 inclusive.

** Pertains to pay grades E-7, 8 and 9 inclusive.

The initial inventory used in evaluating the computer models was obtained from an arbitrarily chosen combination of four specific NEC's, encompassing two separate critical ratings, associated with the Fleet Ballistic Missile program, and representing closely related skills and training. The authors have chosen not to identify the specific NEC's and ratings used in order to preclude the necessity of giving the thesis a military security classification. The initial inventory was compiled from

data obtained from the Bureau of Naval Personnel, using the criteria enumerated previously. Figure II-6 is the standard initial inventory used for all computer runs discussed in this thesis.

PAY GRADE		E-3	E-4		E-5		E-6			E-7			
ENLISTMENT		1	1	2	1	2	1	2	3	2	3	4	5
Years	1	0	5	0	13	3	1	1	0	0	0	4	0
Obligated	2	6	12	0	18	10	0	11	1	1	5	7	0
Service	3	8	33	1	16	12	4	15	9	2	9	17	0
	4	3	13	1	21	13	3	13	13	0	14	18	0
	5	20	20	0	32	12	5	8	3	5	11	4	0
	6	0	48	0	26	2	2	7	4	4	13	2	0

Figure II-6

Standard Initial Inventory

b. Requirements

<u>The Five Year Force Structure and Financial Program</u> is a top secret document by which the Secretary of Defense establishes the approved force structure and financial levels for the various military departments for the succeeding five fiscal years. Emanating from this document are the force levels for each of the approved programs or weapons systems in the budget, including the total cost of military personnel for each

program. Thus each approved program includes the total cost of the personnel sybsystem associated with each particular weapons system. The overall personnel requirements for each approved program are formulated within the budgetary constraints of this document. These requirements are then broken down within each service to determine the numbers of personnel of various skills, qualifications, and experience required.

It is assumed in all the models described in this thesis that the specific requirements are known by pay grade per year for personnel corresponding to the same grouping as the initial inventory. It is noted that these input requirements by pay grade per year are different from the kind of requirements used in the restraint equations of the mathematical model in Chapter II, page 38. The pyramided form of requirements found in the restraint equations are computed from the input requirements discussed in this section. It is these input requirements, of course, that are of primary interest to the personnel planner.

Attempting to determine the requirements corresponding to the group of critical NEC's used for the standard initial inventory in Figure II-6, the authors found that the data furnished by the Bureau of Naval Personnel showed that these particular requirements were broken down into two categories, "supervisors" and "technicians", where supervisors are defined to be personnel of

pay grades E-5 through E-9, inclusive, and technicians to be men in pay grades E-1 through E-4, inclusive. Upon comparing the relative percentage breakdown of the initial inventory by pay grade with the proportional breakdown of actual requirements by supervisors and technicians, an arbitrary percentage distribution of requirements by pay grade was determined. The table in Figure II-7 should help the reader to understand the derivation of this distribution of requirements. Note that each percentage in the table represents the appropriate ratio of personnel in that pay grade to the total of personnel in all pay grades.

Pay Grade	Initial Inventory	Actual BuPers Requirements	Arbitrary Requirements Breakdown
E-7	20.5%	Supervisors 70%	15%
E-6	17.4%		20%
E-5	30.9%		35%
E-4	24.8%	Technicians 30%	25%
E-3	6.4%		5%

Figure II-7

Derivation of Requirements

Since the total requirements per year for five years force structure are known, the requirements by pay grade per year are obtained by applying the arbitrary distribution shown in the right hand column of Figure II-7 to the total requirements per year. Figure II-8 is a tabulation of the standard

requirements by pay grade per year for the personnel grouping used in all computer runs in the thesis.

Pay Grade \ Year	1	2	3	4	5
E-7	76	89	99	106	106
E-6	102	120	133	142	142
E-5	177	210	51	248	248
E-4	128	149	166	178	178
E-3	26	31	33	36	36

Figure II-8

Standard Requirements

c. Attrition Operator

Associated with the passage of each increment of time is the likelihood that a certain fraction of the initial inventory will have been lost or "attrited" due to unforseen circumstances which may be called "acts of God." In order to account for this "unplanned" loss of personnel the attrition operator (α_n) is introduced into the model. This operator will be composed of the summation of specific types of losses and expressed as a percentage reduction of the total inventory per year.

The numbers in parentheses following the various types of losses are the specific item numbers for attrition of active

duty enlisted personnel as defined in NAVPERS 15658, Navy and Marine Corps Military Personnel Statistics [5].

By definition the attrition operator consists of the following types of personnel losses:

(1) Death (#1[illegible])

(2) Physical Disability (#1120)

(3) Discharge by Undesirable Discharge (#1000)

(4) Discharge by Bad Conduct Discharge (#1040)

(5) Discharge by Dishonorable Discharge (#1050)

(6) Discharge by Reason of Dependency or Hardship (#890)

(7) Intra-Navy Transfer (#1250)

(8) Miscellaneous

The actuarial data used to determine the attrition operator matrix should correspond to the same group of personnel in the initial inventory. In reality, it is recognized that this involves a degree of data specificity that does not exist. Thus the user of this model, as a substitute for a complete breakdown of actuarial data by inventory groups, must make certain value judgements or estimates based on the data available in order to apply it to the particular inventory under consideration.

Conceptually it is possible to determine a unique value for each cell of the attrition operator matrix. Because of

the lack of discrimination of basic attrition data with regard to the specific sample in the initial inventory as well as the intuitive idea that an "act of God" may be equally likely to occur at any particular cell of the inventory matrix, a uniform value for the attrition operator was used throughout. The standard value used in all computer runs is $\alpha_n = 1.5\%$, and was determined from data found in reference [5] as follows:

Category	Percentage
Death	0.16
Physical Disability	0.28
Undesirable Discharge	0.43
Bad Conduct Discharge	0.19
Dishonorable Discharge	0.00
Dependency or Hardship	0.09
Intra-Navy Discharge	0.26
Miscellaneous	0.09
Total	1.50%

d. Retirement Operator

A second means by which personnel leave the service is by voluntary retirement, and thus this factor must also be represented in the model. The retirement operator (β_n) is defined to be the percentage of personnel represented in the n^{th} cell of the inventory matrix who are separated through

voluntary retirement by reason of having completed twenty years or more of satisfactory active service.

The values assigned to each cell of the retirement operator matrix are inputs to the model and hence are at the discretion of the user. The retirement operator used throughout this thesis has positive values for only eleven cells of the basic inventory matrix, (i. e., cells numbered 62 through 72), because only these cells represent a total length of service of twenty years or more. The remaining 61 cells, (i. e., numbered 1 through 61), all have a value of zero to represent the fact that voluntary retirement from one of these cells is not possible.

As for the attrition operator, the statistical data available in the Bureau of Naval Personnel for the retirement operator is not sufficiently broken down by NEC or rating to be used directly. It is necessary, therefore, for the personnel planner to make an estimate of the values of the retirement operator matrix for use in this model. The positive values used as the standard for computer runs in this thesis were estimated by the authors, and are tabulated below.

Cell Number	β_n	Approximate Length of Service (years)
62	0.50	20
63	0.25	21
64	0.50	22
65	0.50	23
66	0.50	24
67	0.50	25
68	0.50	26
69	0.50	27
70	0.75	28
71	0.75	29
72	0.75	30

e. Promotion Operator

In any simulation of the enlisted personnel system, the function of promotion, or the process whereby personnel are advanced in seniority level within the structure, must be considered. In the basic model of this thesis, the promotion operator (γ_n) is defined to be the ratio of the number of men promoted to the total number of men represented in the n^{th} cell of the inventory matrix and is expressed as a percentage. The terms "promoted" or "promotion" used herein are specifically limited to advancement from one enlisted pay grade to the next higher pay grade. It is noted that promotion from enlisted to commissioned officer status is, by definition, not considered to be part of the promotion operator, but is considered elsewhere in the model. Specifically, promotion to commissioned status is included in the attrition operator under the subcategory of "intra-Navy transfers".

Because the promotion operator is limited to intra-enlisted promotions, and the inventory matrix compresses pay grades E-7, E-8, and E-9 into one group, (identifying them all as pay grade E-7), the highest promotion considered in the model is that from E-6 to E-7. Thus the cells of the promotion operator matrix which correspond to pay grade E-7 in the inventory matrix, (cells 49 through 72) have a zero value, to indicate that it is not possible to be promoted from one of these cells. Similarly, because the inventory matrix compresses pay grades E-1, E-2 and E-3 into a single group, (identifying them all as pay grade E-3), the lowest promotion considered by the model is that from E-3 to E-4.

As was the case with the other operators, the promotion operator matrix should be determined from statistical data that corresponds to the same sample population as the initial inventory. While it is felt that more definitive data exists in the Bureau of Naval Personnel for the promotion operator than for any other operator, the authors estimated the values used for the standard promotion operator in order to expedite the development of the computer models.

Figure II-9 is the standard promotion operator matrix used for all computer runs to evaluate the models.

Pay Grade		E-3	E-4		E-5		E-6			E-7			
Enlistment		1	1	2	1	2	1	2	3	2	3	4	5
Years Obligated Service	1	1.00	.60	.60	.45	.45	.00	.29	.29	.00	.00	.00	.00
	2	.75	.60	.60	.45	.45	.00	.29	.29	.00	.00	.00	.00
	3	.75	.60	.60	.30	.45	.00	.29	.29	.00	.00	.00	.00
	4	.75	.60	.60	.15	.45	.00	.29	.29	.00	.00	.00	.00
	5	.75	.40	.60	.00	.45	.00	.29	.29	.00	.00	.00	.00
	6	.00	.00	.60	.00	.45	.00	.29	.29	.00	.00	.00	.00

Figure II-9

Standard Promotion Operator Matrix

A further distinction exists between the promotion operator and the other operators. From the planner's standpoint promotion can be thought of as a "controllable" variable, whereas attrition, reenlistment and retirement might be thought of as "uncontrollable" variables. A controllable variable is one which the Bureau of Naval Personnel can vary by means of policy changes; an uncontrollable variable is one whose value is determined primarily by actuarial data, and over which the planner can exercise little or no control.

In a limited sense, retirement and reenlistment might also be considered controllable variables in the personnel system. An example of such a situation might be a period of national emergency in which retirements and enlistments are "frozen", leaving only attrition or act of God loss as the only method by which a person could leave active service.

f. Reenlistment Operator

As each enlisted man approaches the end of his current enlistment he must decide whether to reenlist and thus continue his period of service, or to accept a discharge and leave the armed forces. Any model which simulates a personnel system should provide for this decision process. In this thesis, the reenlistment operator (δ_n) is defined to be the percentage of men represented in the n^{th} cell of the inventory matrix who

remain on active duty for an additional year of service. If the standard values of the reenlistment operator matrix (see Figure II-10) are viewed in conjunction with the structure and function of the model, it can be seen that the reenlistment operator represents a more conventional reenlistment rate, i.e., the ratio of the number of men reenlisting to those men eligible, upon completion of their obligated service.

Pay Grade		E-3	E-4		E-5		E-6			E-7			
Enlistment		1	1	2	1	2	1	2	3	2	3	4	5
Years Obligated Service	1	.254	.254	.856	.254	.856	.254	.902	.950	.950	1.00	1.00	0.00
	2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Figure II-10

Standard Reenlistment Operator Matrix

Note that all cells in rows two through six contain the value of 1.00 to indicate that 100% of the personnel in those cells will remain on active duty for an additional year. Only in row one, which represents personnel with one year of obligated service and who must soon face the reenlistment decision, do values of the promotion operator between 0 and 1 ($0 \leq \delta_n \leq 1$) occur. The percentage of personnel in row one who reenlist and remain in the inventory matrix thus "move" with the passage of a year to a new cell in row six and in a different column representing the appropriate next higher enlistment. In this fashion, then, the reenlistment operators for the cells of row one can be considered to be the effective reenlistment rates for the inventory.

It is conceptually possible to have a unique value for each cell of the reenlistment operator matrix. This seems consistent with the notion that various lengths of enlistments are possible in the enlisted personnel system, such as six year, four year, and minority enlistments of less than four years. Concerning reenlistments, the authors found that for the whole Navy in FY 63, 63.2% of all reenlistments were for a six year period while the remaining 36.8% of the reenlistments were for periods of four years or less. (Only reenlistments made within 24 hours of discharge were considered in these figures.) In order to obviate the complexities arising from trying to

accommodate different lengths of enlistments in the model, it was decided to introduce a simplifying assumption that only six year reenlistments would be permitted. This assumption is considered reasonable and justifiable when it is remembered that most critical personnel subsystems require such lengthy periods of training that a man is required to have a six year service obligation prior to entering the program.

As was the case with the other operators, the data used to determine the reenlistment operator matrix should correspond to the same group of personnel represented in the inventory matrix. The values for the standard reenlistment operator matrix were derived from data found in [6].

2. Outputs

a. Projected Inventory

The projected inventory is the matrix which represents the group of personnel under consideration after the passage of one year. Basically the projected inventory consists of the initial inventory brought forward in time under the operation of the four operator matrices, together with training inputs added to the system in sufficient numbers to ensure that projected inventory meets requirements.

The projected inventory matrix, corresponding

to the inventory matrix in structure, has 72 cells which represent personnel with specific obligated service, pay grade, and current enlistment.

b. Training Input

The training input is the total quantity of personnel added to the particular personnel system under consideration to ensure that the projected inventory meets requirements. The training input is a matrix of 72 cells with the same structure as the initial inventory matrix.

Within the conceptual framework of the model and the definition of the training input, it is possible to add men to the inventory through any of the 72 cells of the training input matrix. It was deemed feasible, however, from the standpoint of practicality to limit the training input to certain cells according to an arbitrarily assumed distribution. For further discussion of the training input distribution see Section 5 of Appendix A.

CHAPTER III

ADVANCED MODELS

A. Two Year Projection

1. Concept

With the construction of the one year model completed, the next step is the construction of a similar model that will perform the same functions over a two year period. It is obvious that "an answer" could be generated by merely cycling the one year model twice, using the solution to the first year of projection as the input to the second cycle. The use of a two year projection model permits decisions as to quantities of the training inputs during both years simultaneously. It can be shown that there exist situations where the requirements can be met at the end of the second year more economically by providing the necessary input in the first year as a lower pay grade, and hence at a lower training cost. This refinement becomes more important with a model which projects over an increasing number of years.

The initial step in construction of the two year model is to develop a set of equations that will project the original inventory for a two year period.

2. Formulation

The projection equations must represent the inventory

of each cell of the inventory matrix at the end of a two year period in terms of particular quantities. These quantities are (a) original inventory, (b) operators, and (c) training inputs for each year.

Using the projection equation as found on page 36 , and applying it to a two year period, the equation states that the inventory at the end of the second year is a function of the inventory at the end of the first year plus the training input during the second year.

$$P(2) = g\left[P(1) \right] + T(2)$$

Similarly, the inventory at the end of the first year is a function of the original inventory plus the training input during the first year.

$$P(1) = f\left[X \right] + T(1)$$

therefore,

$$P(2) = g\left\{ f\left[X \right] + T(1) \right\} + T(2)$$

In this form P(2) is expressed in terms of known inputs (original inventory and operators) plus the unknown training inputs during both years.

A complete derivation of these projection equations can be found in Section la of Appendix B. The basic form of the two year projection equations is:

$$P_n(2) = \begin{cases} \delta_{n-1}\left[1-\gamma_{n-1}\right] C_{n-1}(1) + D_{n-1}(1) + T_n(2) & n \neq 1,7,19,31,49 \\ D_n(1) + T_n(2) & n = 49 \\ T_n(2) & n = 1,7,19,31 \end{cases}$$

where

$$C_n(1) = \begin{cases} \left[1-(\alpha_{n-1}+\beta_{n-1})\right]\left[\delta_{n-2}(1-\gamma_{n-2})\, C_{n-1}(0) + D_{n-1}(0) + T_{n-1}(1)\right] \\ \qquad\qquad\qquad n = 3\text{-}6, 9\text{-}18, 21\text{-}30, 33\text{-}48, 51\text{-}72 \\ \left[1-(\alpha_{n-1}+\beta_{n-1})\right]\left[D_{n-1}(0) + T_{n-1}(1)\right. & n = 50 \\ \left[1-(\alpha_{n-1}+\beta_{n-1})\right] T_{n-1}(1) & n = 2,8,20,32 \\ 0 & n = 1,7,19,31,49 \end{cases}$$

and

$$D_n(1) = \begin{cases} \delta_{n-7}\,\gamma_{n-7}\left[1-(\alpha_{n-7}+\beta_{n-7})\right]\left[\delta_{n-8}(1-\gamma_{n-8})\, C_{n-7}(0)+D_{n-7}(0)+T_{n-7}(1)\right] \\ \qquad\qquad\qquad n = 9\text{-}12 \\ \delta_{n-7}\,\gamma_{n-7}\left[1-(\alpha_{n-7}+\beta_{n-7})\right] T_{n-7}(1) & n = 8 \\ \delta_{n-13}\,\gamma_{n-13}\left[1-(\alpha_{n-13}+\beta_{n-13}\right]\left[\delta_{n-14}(1-\gamma_{n-14})\, C_{n-13}(0)\right. \\ \qquad \left. + D_{n-13}(0) + T_{n-13}(1)\right] \qquad n = 21\text{-}30, 33\text{-}42, 49\text{-}60 \\ \delta_{n-13}\,\gamma_{n-13}\left[1-(\alpha_{n-13}+\beta_{n-13})\right] T_{n-13}(1) & n = 20,32 \\ 0 \qquad n = 1\text{-}7, 13\text{-}19, 31, 43\text{-}48, 61\text{-}72 \end{cases}$$

It may be of some benefit to the reader to appreciate visually the possible moves into and out of each cell of the inventory array during the two year period. This information is shown in Figure III-1

With the projection equations completed the next step is to provide the necessary inputs to a linear program. The linear program will minimize the training input for both years while meeting the requirements for both years. It is necessary then to construct the requirement restraint equations as was done in the one year model.

The reader will recall that in the one year model, Chapter II, the solution to the training input, although originally determined by pay grade totals, needed to be distributed by some rule throughout the inventory matrix in order to represent the projected inventory cell by cell. This distribution now becomes of paramount importance since the projection equations operate on individual cells and therefore require the inventory at the end of each year to be expressable cell by cell.

With an understanding that such a distribution rule is a necessary input to the model, the generation of the restraint equations and hence the necessary inputs to a linear program is possible. A detailed derivation and a listing of these equations and associated linear program inputs can be found in Section la

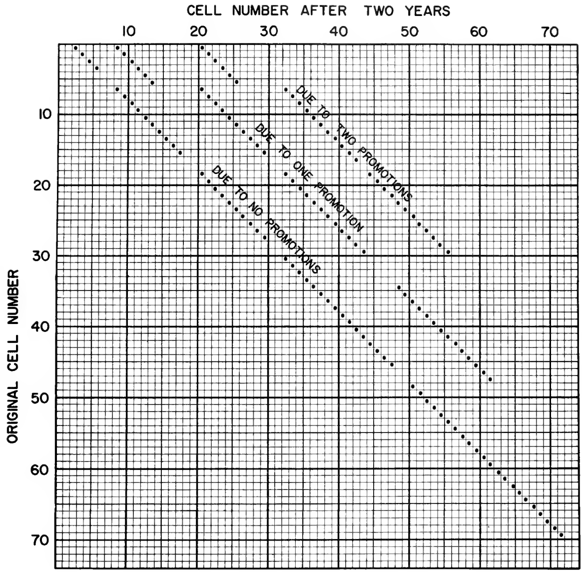


FIGURE III-1 TWO YEAR MOVEMENT GRAPH

of Appendix B.

This linear program will compute the minimum training inputs for both years of projection that are necessary in order to meet the stated requirements for both years.

With the training inputs now available, the projection equations will determine the projected inventory at the end of two years.

B. Five Year Projection Model

1. Concept

With the one and two year models completed, the framework from which a more complex model may be generated is available. In order to accomplish this, one must fully appreciate the iterative nature of the projection equations and the restraint equations as the number of years of projection is increased. In this section it will be shown how the projection equations are formed for the pay grade E-6. The equations for all other pay grades are derived similarly and hence will not be illustrated.

For a two year projection:

$$\sum_{n=31}^{48} P_n(2) = \begin{bmatrix}\text{The number of personnel represented in approp-}\\ \text{riate cells of the initial inventory projected for}\\ \text{two years under the influence of the attrition,}\\ \text{retirement and promotion operators, with \underline{no}}\\ \text{training input.}\end{bmatrix}$$

$$+ \begin{bmatrix}\text{That portion of the first year training input to}\\ \text{E-5 that is promoted to E-6, and not lost from}\\ \text{the inventory due to the retirement, attrition,}\\ \text{or reenlistment operators during the second year.}\end{bmatrix}$$

$$+ \begin{bmatrix}\text{That portion of the first year training input to E-6}\\ \text{that is neither promoted nor lost from the in-}\\ \text{ventory due to the retirement, attrition, or}\\ \text{reenlistment operators during the second year.}\end{bmatrix}$$

$$+ \begin{bmatrix}\text{The training input to E-6 during the second year.}\end{bmatrix}$$

Rewriting this equation in symbols,

$$\sum_{n=31}^{48} P_n(2) = \sum_{n=31}^{48} Y_n(2) + d_1 \tau_3(1) + d_2 \tau_4(1) + \tau_4(2)$$

where,

$Y_n(m)$ = Inventory in cell n after m years, no training input

$\tau_k(m)$ = Total training input to the k^{th} pay grade during the m^{th} year; k = 1, 2, ..., 5

d_i = Constant less than unity; dependent on known promotion, retirement, attrition, and reenlistment operators.

For a three year projection:

$$\sum_{n=31}^{48} P_n(2) =$$

The number of personnel represented in appropriate cells of the initial inventory projected for three years under the influence of the attrition, promotion, retirement and reenlistment operators, with <u>no</u> training input.

\+ Those portions of $d_1 \tau_3(1)$, $d_2 \tau_4(1)$ and $\tau_4(2)$ that are neither promoted nor lost from the inventory due to the retirement, reenlistment or attrition operators during the third year.

\+ That portion of $\tau_3(1)$ that was not promoted during the second year, but was promoted to E-6 during the third year and not lost from the inventory due to the retirement, reenlistment or attrition operators during either the second or third year.

\+ That portion of the second year training input to E-5 that is promoted to E-6 and not lost from the inventory due to the retirement, reenlistment or attrition operators during the third year.

\+ That portion of the first year training input to E-4 that was promoted to E-5 in the second year, promoted to E-6 in the third year, and not lost from

the inventory due to the retirement, reenlistment or attrition operators during either the first, second, or third year.

+ The training input to E-6 during the third year.

and rewriting this equation in symbols,

$$\sum_{n=31}^{48} P_n(2) = \sum_{n=31}^{48} Y_n(3) + d_3 d_1 \tau_3(1) + d_4 d_2 \tau_4(1) + d_5 \tau_4(2)$$

$$+ d_6(1-d_1)\,\tau_3(1) + d_7 \tau_3(2) + d_8 \tau_2(1) + \tau_4(3)$$

These equations illustrate clearly the iterative nature of the projection equations. Since the restraint equations are derived directly from the projection equations, this identical iteration is also found in the restraint equations. The coefficients of the unknowns in the restraint equations are the elements of the matrix of coefficients, which is an input of the linear program model used to minimize the training input needed to meet requirements. If one were to write the five restraint equations for each year of projection, one under the other, with the like training inputs aligned under each other, the coefficients can be said to define the elements of a matrix (q x q) where q equals five times the number of years of projection. This matrix is the matrix of coefficients, and its form depends on the number of years of projection for which the model

is written. Therefore, there exists a different matrix of coefficients, A, for each of the three models discussed in this paper.

Recall that for the one year projection model:

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

where the five rows represent the five restraint equations and the five columns represent the five training inputs during the one year of projection.

Recall that for the two year projection model:

$$
\begin{array}{cc}
 & \begin{array}{cc} \overbrace{\hspace{14em}}^{\text{First Year}} & \overbrace{\hspace{12em}}^{\text{Second Year}} \end{array} \\
A = & \left[\begin{array}{cccccccccc}
1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
a_{7,1} & a_{8,2} & a_{9,3} & a_{10,4} & a_{11,5} & 1 & 1 & 1 & 1 & 1 \\
a_{8,1} & a_{8,2} & a_{9,3} & a_{10,4} & a_{11,5} & 0 & 1 & 1 & 1 & 1 \\
0 & a_{9,2} & a_{9,3} & a_{10,4} & a_{11,5} & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & a_{10,3} & a_{10,4} & a_{11,5} & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & a_{10,5} & a_{11,5} & 0 & 0 & 0 & 0 & 1
\end{array}\right]
\begin{array}{l}
\left.\begin{array}{c} \\ \\ \\ \\ \\ \end{array}\right\}\begin{array}{l}\text{First}\\ \text{Year}\end{array} \\
\left.\begin{array}{c} \\ \\ \\ \\ \\ \end{array}\right\}\begin{array}{l}\text{Second}\\ \text{Year}\end{array}
\end{array}
\end{array}
$$

where the rows represent the five restraint equations for each of two years, and the columns represent the five training inputs for each of the two years.

The important thing to notice is that although the size of the matrix has quadrupled, the number of elements that have to be computed anew represent less than one quarter of the new matrix; in particular, these elements are confined to the lower left five-by-five submatrix. This fact is true for every increase in years of projection; therefore, the problem of determining explicit equations for the elements of the matrix reduces to finding

equations for those elements in the first five columns no matter how many years the model will encompass in its projection. In addition, the reader will note that certain elements are repeated in the submatrix because of the iterative nature of the projection and restraint equations.

Since the complexity of the problem increases with an increase in the number of years of projection, a judicial choice must be made as to the optimal number of years for which the model should be designed. The usefulness of the model is limited to the number of years for which the requirements are stated. The Force Requirements are currently precisely stated for five years and hence the optimal model is a five year model.

2. Formulation

The complete derivation of the necessary inputs to the linear program for the five year model is of such length and detail that it is not shown in this paper. The explicit expressions for each input in terms of the known operators and original inventory can be found in Section 2b of Appendix B. These expressions were derived using the same training input distribution as was used in the basic model.

The output of the linear program consists of the minimum training inputs over the five years that will meet requirements for

each of the five years. With these training inputs available, the projection equations can be used to express the projected inventory at the end of each of the five years.

C. Inputs - Outputs

1. Inputs

The following inputs of the advanced models are defined the same as in the basic model (see Section C, Chapter II) and hence will not be discussed in this section:

a. Initial Inventory

b. Requirements

c. Attrition Operator

d. Retirement Operator

e. Promition Operator

f. Reenlistment Operator

There are two additional inputs to the advanced models which were not defined in the basic model. The first of these two inputs is the training input distribution, which by definition is the arbitrary designation of those cells in the inventory matrix which can receive a training input. Thus the training input distribution determines which cells of the training input matrix will have positive values and which ones will have a zero value. The reader will note that the linear programming solution determines the values of the training inputs while the training input distribution determines the location of these values in the inventory matrix.

Implicit in the definition of training input distribution is the assumption that the total training input by pay grade per year ($\tau_k(m)$) will be divided uniformly among the designated cells of each particular pay grade.

The training input distribution used for the computer runs of the advanced models are discussed in Section 5 , Appendix A.

The second of the additional inputs to the advanced models are the cost coefficients. As defined in this thesis, the cost coefficients are algebraic factors which represent costs associated with the training of personnel of different pay grades. These cost coefficients appear in the statement of the objective function of the linear program, which determines the optimal training input with regard to minimum total cost of training. The objective function for the advanced models is to

$$\text{minimize} \quad \sum_{m=1}^{j} \sum_{k=1}^{5} c_k \, \tau_k(m)$$

where $j = 2$ for the two year model

$j = 5$ for the five year model

c_k = cost coefficient for the k^{th} pay grade

$\tau_k(m)$ = total training input to the k^{th} pay grade in the m^{th} year

The cost coefficients provide the planner with a means for quantitatively expressing the complex relationships among training costs for personnel of various pay grades. The cost coefficients may take on any appropriate units, and thus are not restricted to the conventional units of "dollars" normally associated with the traditional concept of cost. For example, the planner might desire to let the cost coefficients represent a measure of the difficulty of procuring personnel for a specific training program, with appropriate units.

For the computer runs of the advanced models presented in the thesis, it was desired to obtain cost coefficients which were relatively simple in concept and yet quantifiable in some sense in order that some significance could be attached to the linear programming solution. Under the assumptions that (1) the fixed overhead costs of operating a Navy school are constant regardless of who the students are, and (2) that personnel in training are in a so-called non-productive status relative to the rest of the Navy, a possible measure of the relative training costs per pay grade would be the ratios of the various trainees' pay scales. For example, it costs more to train a chief petty officer than it does a seaman in a given unit of time, because the chief's basic pay is grater than the seaman's while both are in the "non-productive" training status.

The standard cost coefficients used in the computer runs of the advanced models were derived by taking the ratio of the basic pay of each enlisted pay grade to the basic pay of a man in pay grade E-3. The basic pay corresponding to the mean length of service for each pay grade was the criterion used to select specific entries from the enlisted pay table. The cost coefficients per pay grade determined in this manner are listed below:

Pay Grade E-3	$c_1 = 1.00$
Pay Grade E-4	$c_2 = 1.32$
Pay Grade E-5	$c_3 = 1.58$
Pay Grade E-6	$c_4 = 1.87$
Pay Grade E-7	$c_5 = 2.32$

2. Outputs

The following outputs of the advanced models are defined the same as in the basic model (see Section C, Chapter II) and are:

a. Projected Inventory

b. Training Input

CHAPTER IV

DIGITAL COMPUTER MODELS

A. General

The digital computer programs of the mathematical models presented in Chapters II and III were written in FORTRAN 60 computer language, and all computer runs were made on the U. S. Naval Postgraduate School's Control Data Corporation (CDC) 1604 high-speed digital computer. All of the programming for these models was done by the authors except for the linear programming subroutines (i. e., subroutines "LINEAR" and "SIMPLEX") which are standard routines on the library tape of the Naval Postgraduate School Computer Center.

The manner in which the computer programs were written was such as to insure that the following objectives are satisfied:

1. To provide programs that are user oriented. The programs are designed to provide maximum flexibility and adaptability for the user, and to facilitate his understanding and control of the input data as well as his analysis of the resultant outputs.

2. To provide within each model the capability of projecting an initial inventory and computing the least expensive training inputs for a period of five years.

3. To demonstrate the capabilities of the computer models through the use of actual data inputs.

4. To write the programs using notation that is consistent, insofar as practicable, with the notation and symbology used in the development of the mathematical models.

5. To write the programs using subroutines, or separate "packages" which accomplish certain specific operations or functions common to all models.

The authors are satisfied that each of these objectives has been fulfilled.

To distinguish the computer programs for the convenience of the reader, the following designations will apply to the models previously discussed:

Model	Designation
ONE-YEAR PROJECTION	PIPE ONE
TWO-YEAR PROJECTION	PIPE TWO
FIVE-YEAR PROJECTION	PIPE FIVE

Program listings for PIPE ONE and PIPE FIVE are found in Appendix E. The listing for PIPE TWO is not included in this thesis because PIPE TWO was written only to facilitate the development of the five-year model, PIPE FIVE. If only a two-year projection run is desired, of course, the user need only use the first and second year of the five-year solution as generated by the program for PIPE FIVE.

In the remaining sections of this chapter, functional flow diagrams of the individual programs will be presented in order to provide the reader with the underlying logic used in the development

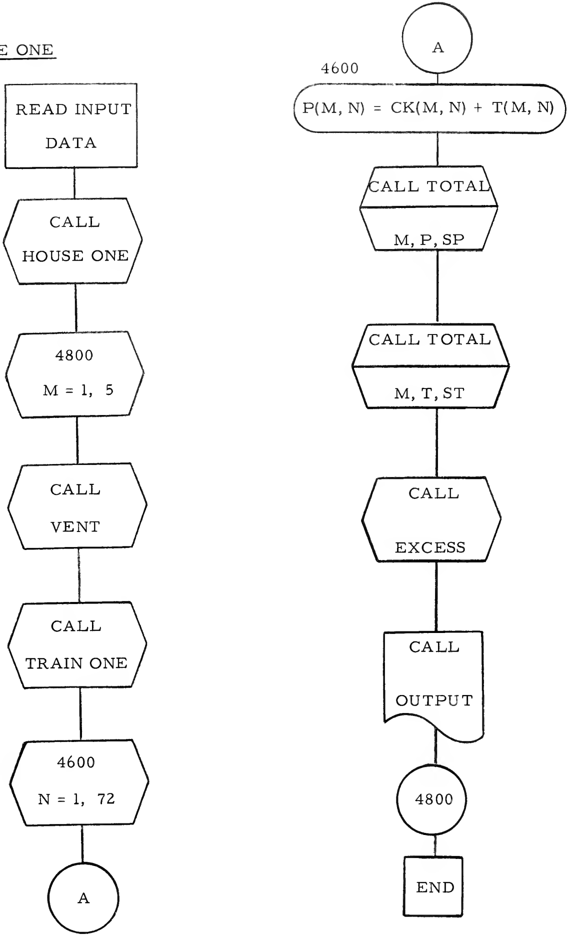
of the respective computer programs. The computer runs used to illustrate these programs were made with the standard input values (discussed in Section C, Chapter I and Section C, Chapter II) in order to provide a common basis for a quantitative analysis of the solutions as generated by PIPE ONE and PIPE FIVE.

B. FUNCTIONAL FLOW DIAGRAMS

PIPE ONE and PIPE FIVE are constructed from a series of subroutines. The function of these individual subroutines will be discussed in Section C of this chapter. The actual program listing of these programs may be found in APPENDIX E.

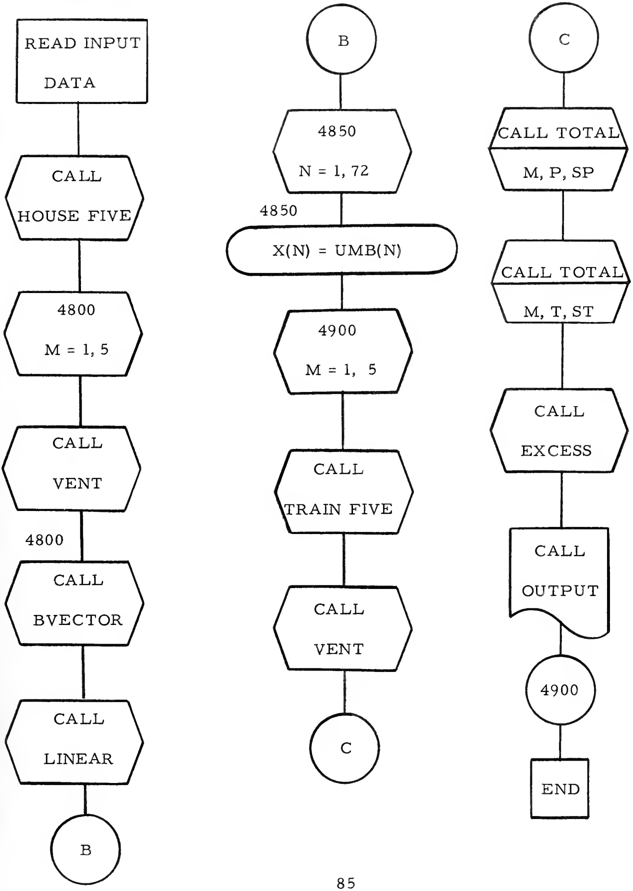
The following illustrations are functional flow diagrams of PIPE ONE and PIPE FIVE:

PIPE ONE



READ INPUT DATA
CALL HOUSE ONE
4800 M = 1, 5
CALL VENT
CALL TRAIN ONE
4600 N = 1, 72
A
A
4600
P(M, N) = CK(M, N) + T(M, N)
CALL TOTAL
M, P, SP
CALL TOTAL
M, T, ST
CALL EXCESS
CALL OUTPUT
4800
END

PIPE FIVE



READ INPUT DATA
CALL HOUSE FIVE
4800 M = 1, 5
CALL VENT
4800
CALL BVECTOR
CALL LINEAR
B
B
4850 N = 1, 72
4850
X(N) = UMB(N)
4900 M = 1, 5
CALL TRAIN FIVE
CALL VENT
C
C
CALL TOTAL M, P, SP
CALL TOTAL M, T, ST
CALL EXCESS
CALL OUTPUT
4900
END

C. SUBROUTINES

Each of the subroutines used in building PIPE ONE and PIPE FIVE are discussed in the following paragraphs.

1. HOUSE ONE: This subroutine is used only with PIPE ONE. Its function is to compute those constants and other values which are needed in the iterative routines of the program but do not themselves change during a projection. Specifically this routine:

   a. automatically sets a zero value for BETA, retirement operator, in all cells below 62;

   b. sets a value of one in all cells of DELTA, re-enlistment operator, except for the cells in row one;

   c. converts requirements by pay grade into the comulative requirements form used in the restraint equations;

   d. calls the subroutine FACTOR.

2. HOUSE FIVE: This subroutine is used only with PIPE FIVE and performs the same functions as HOUSE ONE and in addition:

   a. computes the constants EAl, UA, VA, and WA;

   b. sets the initial value of each cell of the A-matrix to zero;

c. sets the values of the cost coefficients into the first row of the A-matrix where they form the objective function to be used in subroutine LINEAR;

d. calls the subroutine AMAT.

3. AMAT: This subroutine produces the values of the elements of the A-matrix of the linear program. It makes use of subroutines SUM and FIX to solve for the values of the a_{ij} found in the mathematical formulation Appendix B, section 2, "Advance Models."

4. FACTOR: This subroutine computes for each pay grade the divisor for the training input so that the computed training input may be distributed uniformly over the cells designated for training input.

5. VENT: This subroutine solves the basic one year projection equations found in Chapter II. It is the basic projection subroutine of both PIPE ONE and PIPE FIVE. In both programs, VENT is used in an iterative manner and is cycled as many times as there are years of projection. It uses as a starting inventory the end-of-year inventory of the year before.

6. BVECTOR: This subroutine produces the B-vector for the linear program. It solves the following equation for B_i:

$B_i = R_{m,k}$ - (that portion of the initial inventory still on board in the $m\underline{th}$ year corresponding to the $k\underline{th}$ pay grade)

7. LINEAR and SIMPLEX: These routines solve the linear programming problem for the optimum training input by pay grade and compute the cost of this input.

8. TRAIN ONE: This subroutine computes the training input for PIPE ONE and then distributes this input to the appropriate cells of the inventory.

9. TRAIN FIVE: This subroutine takes the training input by pay grade as computed by LINEAR and distributes it according to the training input distribution to the appropriate cells of the training input matrix. (VENT is then called once again and the training input is added in the proper cell at the appropriate time in the projection.)

10. TOTAL: This subroutine provides a means of adding the cells of the inventory matrix and training matrix for use in the subroutine OUTPUT.

11. EXCESS: This subroutine computes any excesses and shortages of requirements for use by subroutine OUTPUT.

12. OUTPUT: This subroutine causes the output to be printed in the form shown in Appendix D.

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APPENDICES

APPENDIX A

ASSUMPTIONS

This Appendix discusses eight major assumptions made by the authors in the development of PIPE.

1. It is assumed that the inventories to which PIPE is applicable are critical subsystems of the enlisted personnel structure of the Navy. This assumption was made because the critical subsystems presently require the immediate consideration of the personnel planner, and these subsystems permit a simpler formulation of the inventory matrix.

2. The selection of which enlistments are to be represented in each pay grade of the inventory matrix is based upon the assumption that the inventories considered be those of critical personnel subsystems of the Navy. An analysis of the current inventories of a number of these subsystems indicated that the numbers of personnel in the other than the following categories were negligible:

Pay Grade	Enlistment
E-3	1st
E-4	1st, 2nd
E-5	1st, 2nd
E-6	1st, 2nd, 3rd
E-7	2nd, 3rd, 4th, 5th

The inventory matrix was therefore restricted to represent only those personnel in the listed categories.

3. The personnel requirements by enlisted pay grade per year of projection are assumed to be available to the user. Although the requirements in this form are inputs to PIPE, the requirements used in the restraint equations of the linear program are various sums of these input requirements. This is done so as to insure that the requirements for the senior pay grades are filled before those of the junior pay grades.

4. Since PIPE projects critical personnel subsystems, it is assumed that there is an unlimited supply of personnel to fill the requirements of these subsystems. This means that no matter how large a training input PIPE determines as necessary, the priority enjoyed by the subsystem assures that these personnel will be provided from the balance of the personnel system. This is admittedly a program of suboptimization within the personnel system of the Navy; however, it does appear to be a realistic approximation of current policy.

5. The construction of the inventory matrix requires the assumption of a method by which the training input will be distributed among the cells represented by any one pay grade. PIPE has been constructed to distribute the training input uniformly among

a pre-selected number of cells within each pay grade. The particular distribution selected by the authors is illustrated in Figure A-1.

6	6	12	18	24	30	36	42	48	54	60	66	72
5	5	11	17	23	29	35	41	47	53	59	65	71
4	4	10	16	22	28	34	40	46	52	58	64	70
3	3	9	15	21	27	33	38	45	51	57	63	69
2	2	8	14	20	26	32	39	44	50	56	62	68
1	1	7	13	19	25	31	37	43	49	55	61	67

= equal input within pay grade

= no input

FIGURE A-1

STANDARD TRAINING INPUT DISTRIBUTION

The reasons for the omission of some cells are:

a. Cell numbers (1, 7, 13, 19, 25, 31, 37, 43, 49, 55, 61, 67). These cells represent the first year of an enlistment and this would be the year spent in schools preparing the personnel for input into the program.

b. Cell numbers (6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72). These cells represent the last year of an enlistment and it is doubtful that personnel would be sent to school so late in an enlistment that they had but one year obligated service upon return to the operating forces.

c. Cell numbers (8, 17, 32-35, 62-65, 68-71).

These cells are considered very unlikely to have training.

PIPE is designed so as to provide the user with the ability to select the cells into which training input is to be permitted. The only restriction to the user's choice of distribution is that PIPE will automatically distribute the training input to each pay grade uniformly among the cells selected by the user.

6. The manner in which training inputs are used to construct the objective function of the linear program demands consideration of the fact that the cost to the Navy is different for the procurement of personnel from the senior pay grades than from the lower pay grades. This implies that some type of cost coefficient must be used in the objective function of the linear program. It is also recognized that although this difference in cost is important and must be considered, there was insufficient time available to the authors to analyze any data that may be recorded on this subject. Therefore, to include this difference in cost, PIPE uses cost coefficients based solely on relative enlisted pay scales. The manner in which these cost coefficients are computed is as follows:

a. Compute the average base pay for each of the five pay grades of the inventory array

average base pay for E-3	=	PE3
average base pay for E-4	=	PE4
average base pay for E-5	=	PE5
average base pay for E-6	=	PE6
average base pay for E-7, 8, 9	=	PE79

b. Divide each average pay by the average pay of the pay grade E-3.

$$C_1 = \frac{PE3}{PE3} = 1$$

$$C_2 = \frac{PE4}{PE3}$$

$$C_3 = \frac{PE5}{PE3}$$

$$C_4 = \frac{PE6}{PE3}$$

$$C_5 = \frac{PE79}{PE3}$$

where

C_i = cost coefficients $i = 1, 2, \ldots 5$

Although these coefficients do not fully express the difference in procurement costs among pay grades, they are considered to be satisfactory until such time as a cost analysis is conducted and more sophisticated data is available.

The cost coefficients are assumed to be the same for all years included in the inventory projection. Since these coefficients are inputs to PIPE, the user may change their values in any manner he chooses.

7. The values assigned to the promotion, retirement, attrition, and reenlistment operators used in PIPE are assumed to be invariant during the period of projection. If the user desired to vary these values the symbols representing these operators must be additionally subscripted to provide for the designation by year, and the mathematical equations altered to reflect this sophistication.

8. It is assumed that all personnel that reenlist do so for six years, and the time at which they reenlist is restricted to the normal expiration of current enlistment. Therefore neither early nor broken service reenlistments are considered in PIPE.

APPENDIX B

MATHEMATICAL FORMULATION

I BASIC MODELS

a. Derivation of Inputs to the Linear Program

1. Requirements Specified for Every Cell

Since α, β, γ, δ, and X are all inputs to the model and hence known quantities for $n = 1,2,\ldots,72$, it follows that

(1) $$P_n(1) = T_n(1) + CK_n(1) \qquad n = 1,2,\ldots,72$$

where $CK_n(1)$ is a function of only the known quantities enumerated above.

The requirements state that

(2) $$P_n(1) \geq R_n(1) \qquad n = 1,2,\ldots,72$$

where $R_n(1) \equiv$ requirement for cell n for the first year of projection.

(3) $$\therefore T_n(1) + CK_n(1) \geq R_n(1) \qquad n = 1,2,\ldots,72$$

or $$T_n(1) \geq R_n(1) - CK_n(1) \qquad n = 1,2,\ldots,72$$

(4) Let $$B_n(1) = R_n(1) - CK_n(1) \qquad n = 1,2,\ldots,72$$

(5) $$\therefore \quad T_n(1) \geq B_n(1) \qquad n = 1,2,\ldots,72$$

This yields a set of 72 independent equations, and hence a 72 by 72 A matrix, a 72 by 1 B matrix, a 1 by 72 c matrix, and a 72 by 72 cY matrix. This procedure is not pursued further.

2. Requirements Specified for Each Pay Grade

(6) $$P_n(1) = T_n(1) + CK_n(1) \qquad n = 1,2,\ldots,72$$

The requirements state that

(7a) $$\sum_{n=1}^{72} P_n(1) \geq R_1(1)$$

(7b) $$\sum_{n=7}^{72} P_n(1) \geq R_2(1)$$

(7c) $$\sum_{n=19}^{72} P_n(1) \geq R_3(1)$$

(7d) $$\sum_{n=31}^{72} P_n(1) \geq R_4(1)$$

(7e) $$\sum_{n=49}^{72} P_n(1) \geq R_5(1)$$

where the $R_k(1)$, ($k = 1,2,\ldots,5$) are as defined in Chapter II.

Substituting equation (6) into equations (7a-7e), we have

(8a) $$\sum_{n=1}^{72} T_n(1) \geq R_1(1) - \sum_{n=1}^{72} CK_n(1) = B_2$$

(8b) $$\sum_{n=7}^{72} T_n(1) \geq R_2(1) - \sum_{n=7}^{72} CK_n(1) = B_3$$

(8c) $$\sum_{n=19}^{72} T_n(1) \geq R_3(1) - \sum_{n=19}^{72} CK_n(1) = B_4$$

(8d) $$\sum_{n=31}^{72} T_n(1) \geq R_4(1) - \sum_{n=31}^{72} CK_n(1) = B_5$$

(8e) $$\sum_{n=49}^{72} T_n(1) \geq R_5(1) - \sum_{n=49}^{72} CK_n(1) = B_6$$

where the B_i $(i = 1, 2, \ldots, 5)$ can be computed since

$$CK_n(1) = \begin{cases} 0 & n = 1, 7, 19, 31 \\ \delta_{n-1}\left[1-\gamma_{n-1}\right]\left[1-(\alpha_{n-1}+\beta_{n-1})\right] X_{n-1} & n = 2\text{-}6, 14\text{-}18, 44\text{-}48, 62\text{-}72 \\ \delta_{n-1}\left[1-\gamma_{n-1}\right]\left[1-(\alpha_{n-1}+\beta_{n-1})\right] X_{n-1} + \delta_{n-7}\gamma_{n-7}\left[1-(\alpha_{n-7}+\beta_{n-7})\right] X_{n-7} & \\ & n = 8\text{-}13 \\ \delta_{n-1}\left[1-\gamma_{n-1}\right]\left[1-(\alpha_{n-1}+\beta_{n-1})\right] X_{n-1} + \delta_{n-13}\gamma_{n-13}\left[1-(\alpha_{n-13}\right. & \\ \qquad \left.+\beta_{n-13})\right] X_{n-13} & n = 20\text{-}30, 32\text{-}43, 49\text{-}61 \\ \delta_{n-13}\gamma_{n-13}\left[1-(\alpha_{n-13}+\beta_{n-13})\right] X_{n-13} & n = 49 \end{cases}$$

Define $\tau(m) = \left[\tau_k(m)\right]$ where $\tau_k(m) \equiv$ the total training input for the k^{th} pay grade during the m^{th} year of projection.

(9a) $\therefore$ $$\tau_1(1) = \sum_{n=1}^{6} T_n(1)$$

(9b) $$\tau_2(1) = \sum_{n=7}^{18} T_n(1)$$

(9c) $$\tau_3(1) = \sum_{n=19}^{30} T_n(1)$$

(9d) $$\tau_4(1) = \sum_{n=31}^{48} T_n(1)$$

(9e) $$\tau_5(1) = \sum_{n=49}^{72} T_n(1)$$

Therefore

(10a) $$\sum_{n=1}^{72} T_n(1) = \sum_{k=1}^{5} \tau_k(1)$$

(10b) $$\sum_{n=7}^{72} T_n(1) = \sum_{k=2}^{5} \tau_k(1)$$

(10c) $$\sum_{n=19}^{72} T_n(1) = \sum_{k=3}^{5} \tau_k(1)$$

(10d) $$\sum_{n=31}^{72} T_n(1) = \sum_{k=4}^{5} \tau_k(1)$$

(10e) $$\sum_{n=49}^{72} T_n(1) = \tau_5(1)$$

Therefore the linear restraint equations are:

(11a) $$\tau_1(1) + \tau_2(1) + \tau_3(1) + \tau_4(1) + \tau_5(1) \geq B_2$$

(11b) $$\tau_2(1) + \tau_3(1) + \tau_4(1) + \tau_5(1) \geq B_3$$

(11c) $$\tau_3(1) + \tau_4(1) + \tau_5(1) \geq B_4$$

(11d) $$\tau_4(1) + \tau_5(1) \geq B_5$$

(11e) $$\tau_5(1) \geq B_6$$

These five linear restraint equations produce the following necessary inputs to the linear program:

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \end{bmatrix}$$

$c = (c_1, c_2, c_3, c_4, c_5)$ as defined in Chapter II.

$Y = \tau(1) = \left[\tau_k(1) \right]$

and the objective function is

$$\text{minimize } cY = \min \sum_{k=1}^{5} c_k \tau_k(1)$$

The output of the linear program is the matrix $\tau(1)$, the elements of which are the total training input for each of the five pay grades. These totals represent the minimum training input to each of thepay grades that will produce a projected inventory that satisfies the stated requirements at the end of one year of projection.

$$\text{Recall, } \tau_k(1) = \sum_{n \epsilon k} T_n(1)$$

In order to represent each cell of the projected inventory, $P_n(1)$, it is necessary to determine $T_n(1)$ explicitly for each $n = 1, 2, \ldots, 72$. This demands some rule by which the total training input to any one pay grade, $\tau_k(1)$, be distributed among the cells within that pay grade. A discussion of this problem and a suitable solution is explained in detail in Appendix A . Using this rule, each $T_n(1)$ $(n = 1, 2, \ldots, 72)$ can be determined once having solved for the $\tau_k(1)$ $(k = 1, 2, \ldots, 5)$. Knowing the value for each $T_n(1)$, the projection equations, (the only unknowns of which are the $T_n(1)$), can then be used to explicitly state each of the $P_n(1)$ and hence the entire projected inventory after one year, $P(1)$.

APPENDIX B

MATHEMATICAL FORMULATION

II. ADVANCED MODELS

a. Two Year Model

This section of Appendix B contains the mathematical formulation of the two year model discussed in Section A of Chapter III.

1. Derivation of Projection Equations

Applying the projection equation of the basic model (Chapter II, page 36) to a two year period, the inventory at the end of the second year of projection is expressed as:

$$(12)\quad P_n(2) = \begin{cases} \delta_{n-1}(1-\gamma_{n-1})\, C_n(1) + D_n(1) + T_n(2) & n \neq 1,7,19,31,49 \\ D_n(1) + T_n(2) & n=49 \\ T_n(2) & n=1,7,19,31 \end{cases}$$

In these equations $P_n(2)$ is a function of $C_n(1)$, $D_n(1)$ and $T_n(2)$. In order to express $P_n(2)$ as a function of known quantities and the unknown training inputs, it is necessary that $P_n(2)$ be expressed as a function of $C_n(0)$, $D_n(0)$, $T_n(1)$, and $T_n(2)$. The projection equations for the basic model can again be used, but now to express $C_n(1)$ and $D_n(1)$ in the desired terms as follows:

First, $C_n(1)$ can be expressed as

$$(13)\quad C_n(1) = \begin{cases} \left[1-(\alpha_{n-1}+\beta_{n-1})\right] P_{n-1}(1) & n \neq 1,7,19,31,49 \\ 0 & n=1,7,19,31,49 \end{cases}$$

where

$$(14)\quad P_{n-1}(1) = \begin{cases} \delta_{n-2}\left[1-\gamma_{n-2}\right]C_{n-1}(0) + D_{n-1}(0) + T_{n-1}(1) & n\neq 2,8,20,32,50 \\ D_{n-1}(0) + T_{n-1}(1) & n=20-30,32-43,49-61 \\ T_{n-1}(1) & n=1-7,14-19,31,44-48,62-72 \end{cases}$$

Second, $D_n(1)$ can be expressed as

$$(15)\quad D_n(1) = \begin{cases} \delta_{n-7}\gamma_{n-7}\, C_{n-6}(1) & n=8-13 \\ \delta_{n-13}\gamma_{n-13}\, C_{n-12}(1) & n=20-30,32-43,49-61 \\ 0 & n=1-7,14-19,31,44-48,62-72 \end{cases}$$

where we express $C_{n-6}(1)$ and $C_{n-12}(1)$ as

$$(16)\quad C_{n-6}(1) = \begin{cases} \left[1-(\alpha_{n-7} + \beta_{n-7})\right]P_{n-7}(1) & n\neq 7,13,25,37,55 \\ 0 & n=7,13,25,37,55 \end{cases}$$

and

$$(17)\quad C_{n-12}(1) = \begin{cases} \left[1-(\alpha_{n-13} + \beta_{n-13})\right]P_{n-13}(1) & n\neq 13,19,31,43,61 \\ 0 & n=13,19,31,43,61 \end{cases}$$

and then express $P_{n-7}(1)$ and $P_{n-13}(1)$ as

$$(18)\quad P_{n-7}(1) = \begin{cases} \delta_{n-8}\left[1-\gamma_{n-8}\right]C_{n-7}(0) + D_{n-7}(0) + T_{n-7}(1) & n\neq 8,14,26,38,56 \\ D_{n-7}(0) + T_{n-7}(1) & n=56 \\ T_{n-7}(1) & n=8,14,26,38 \end{cases}$$

and

$$(19)\quad P_{n-13}(1) = \begin{cases} \delta_{n-14}\left[1-\gamma_{n-14}\right]C_{n-13}(0) + D_{n-13}(0) + T_{n-13}(1) & n\neq 14,20,32,44,62 \\ D_{n-13}(0) + T_{n-13}(1) & n=62 \\ T_{n-13}(1) & n=14,20,32,44 \end{cases}$$

A graphical representation of this breakdown is:

$$
\begin{tikzcd}
 & (12) \arrow[dl] \arrow[dr] & & \\
(13) \arrow[d] & & (15) \arrow[dl] \arrow[dr] & \\
(14) & (16) \arrow[d] & & (17) \arrow[d] \\
 & (18) & & (19)
\end{tikzcd}
$$

Equations (13) through (19) serve to express the quantities $C_n(1)$ and $D_n(1)$ of equation (12) in terms of known inputs and training inputs. Retracing these steps so as to ultimately be able to write equation (12) in the desired terms, the procedure is:

Substitute equation (19) into equation (17) which yields

$$
\text{(17a)} \quad C_{n-12}(1) = \begin{cases}
\left[1-(\alpha_{n-13} + \beta_{n-13})\right]\left[\delta_{n-14}(1-\gamma_{n-14})C_{n-13}(0) + D_{n-13}(0) + T_{n-13}(1)\right] & \\
\qquad n \neq 13\text{-}14, 19\text{-}20, 31\text{-}32, 43\text{-}44, 61\text{-}62 & \\
\left[1-(\alpha_{n-13} + \beta_{n-13})\right]\left[D_{n-13}(0) + T_{n-13}(1)\right] & n = 62 \\
\left[1-(\alpha_{n-13} + \beta_{n-13})\right]\left[T_{n-13}(1)\right] & n = 14, 20, 32, 44 \\
0 & n = 13, 19, 31, 43, 61
\end{cases}
$$

Then substitute equation (18) into equation (16) which yields

$$
\text{(16a)} \quad C_{n-6}(1) = \begin{cases}
\left[1-(\alpha_{n-7} + \beta_{n-7})\right]\left[\delta_{n-8}(1-\gamma_{n-8})C_{n-7}(0) + D_{n-7}(0) + T_{n-7}(1)\right] & \\
\qquad n \neq 7, 8, 13, 14, 25, 26, 37, 38, 55, 56 & \\
\left[1-(\alpha_{n-7} + \beta_{n-7})\right]\left[D_{n-7}(0) + T_{n-7}(1)\right] & n = 56 \\
\left[1-(\alpha_{n-7} + \beta_{n-7})\right]T_{n-7}(1) & n = 8, 14, 26, 38 \\
0 & n = 7, 13, 25, 37, 55
\end{cases}
$$

Then substitute equations (16a) and (17a) into (15) which yields

$$(15a)\quad D_n(1) = \begin{cases} \delta_{n-13}\gamma_{n-13}\left[1-(\alpha_{n-13}+\beta_{n-13})\right]\left[\delta_{n-14}(1-\gamma_{n-14})C_{n-13}(0) + D_{n-13}(0) + T_{n-13}(1)\right] & n = 21\text{-}30, 33\text{-}42, 49\text{-}60 \\ \delta_{n-13}\gamma_{n-13}\left[1-(\alpha_{n-13}+\beta_{n-13})\right]T_{n-13}(1) & n = 20, 32 \\ \delta_{n-7}\gamma_{n-7}\left[1-(\alpha_{n-7}+\beta_{n-7})\right]\left[\delta_{n-8}(1-\gamma_{n-8})C_{n-7}(0) + D_{n-7}(0) + T_{n-7}(1)\right] & n = 9\text{-}12 \\ \delta_{n-7}\gamma_{n-7}\left[1-(\alpha_{n-7}+\beta_{n-7})\right]T_{n-7}(1) & n = 8 \\ 0 & n = 1\text{-}7, 13\text{-}19, 31, 43\text{-}48, 61\text{-}72 \end{cases}$$

Then substitute equation (14) into equation (13) which yields

$$(13a)\quad C_n(1) = \begin{cases} \left[1-(\alpha_{n-1}+\beta_{n-1})\right]\left[\delta_{n-2}(1-\gamma_{n-2})C_{n-1}(0) + D_{n-1}(0) + T_{n-1}(1)\right] & n \neq 2, 8, 20, 32, 50 \\ \left[1-(\alpha_{n-1}+\beta_{n-1})\right]\left[D_{n-1}(0) + T_{n-1}(1)\right] & n = 50 \\ \left[1-(\alpha_{n-1}+\beta_{n-1})\right]\left[T_{n-1}(1)\right] & n = 2, 8, 20, 32 \end{cases}$$

Equations (13a) and (15a) along with equation (12) are therefore the projection equations for the two year model. Given an initial inventory and operator matrices, these projection equations will describe the inventory at the end of the second year of a two year projection, in terms of the training input during both years.

2. Derivation of Inputs to the Linear Program

Stating the requirements in the same manner as in equation (7), (page 99), but for both years of projection:

(20a) $$\sum_{n=1}^{72} P_n(1) \geq R_1(1)$$

(20b) $$\sum_{n=7}^{72} P_n(1) \geq R_2(1)$$

(20c) $$\sum_{n=19}^{72} P_n(1) \geq R_3(1)$$

(20d) $$\sum_{n=31}^{72} P_n(1) \geq R_4(1)$$

(20e) $$\sum_{n=49}^{72} P_n(1) \geq R_5(1)$$

(20f) $$\sum_{n=1}^{72} P_n(2) \geq R_1(2)$$

(20g) $$\sum_{n=7}^{72} P_n(2) \geq R_2(2)$$

(20h) $$\sum_{n=19}^{72} P_n(2) \geq R_3(2)$$

(20i) $$\sum_{n=31}^{72} P_n(2) \geq R_4(2)$$

(20j) $$\sum_{n=49}^{72} P_n(2) \geq R_5(2)$$

The restraint equations generated from equations (20a) through (20e) are identical to the five restraint equations of the one year model, equations (11a) through (11e). The problem reduces then to the derivation of the five additional restraint equations represented by equations (20f) through (20j). In order to facilitate the derivation the following combinations of operators and original inventory are defined:

$$G1_n = \delta_{n-7}\gamma_{n-7}$$

$$G2_n = \delta_{n-13}\gamma_{n-13}$$

$$E1_n = \left[1-(\alpha_{n-1} + \beta_{n-1})\right]$$

$$E2_n = \left[1-(\alpha_{n-7} + \beta_{n-7})\right]$$

$$E3_n = \left[1-(\alpha_{n-13} + \beta_{n-13})\right]$$

$$CK1_n = \left[\delta_{n-2}(1-\gamma_{n-2})C_{n-1}(0) + D_{n-1}(0)\right]$$

$$CK2_n = \left[\delta_{n-8}(1-\gamma_{n-8})C_{n-7}(0) + D_{n-7}(0)\right]$$

$$CK3_n = \left[\delta_{n-14}(1-\gamma_{n-14})C_{n-13}(0) + D_{n-13}(0)\right]$$

$$M_n = \delta_{n-1}\left[1-\gamma_{n-1}\right]$$

$$U_n = M_n E1_n$$

$$V_n = G1_n E2_n$$

$$W_n = G2_n E3_n$$

$$Y_n = \delta_{n-1} E1_n$$

The training inputs are defined as in equation (9) but for both years of projection.

(21a) $$\tau_1(m) = \sum_{n=1}^{6} T_n(m) \qquad m = 1,2$$

(21b) $$\tau_2(m) = \sum_{n=7}^{18} T_n(m) \qquad m = 1,2$$

(21c) $$\tau_3(m) = \sum_{n=19}^{30} T_n(m) \qquad m = 1,2$$

(21d) $$\tau_4(m) = \sum_{n=31}^{48} T_n(m) \qquad m = 1,2$$

(21e) $$\tau_5(m) = \sum_{n=49}^{72} T_n(m) \qquad m = 1,2$$

It is at this point that some rule must be assumed for the distribution of the training input throughout each pay grade. The distribution assumed is discussed in Appendix A and generates the following definitions:

(22a) $T_n(m) = 1/4\ \tau_1(m) \qquad m = 1,2 \qquad n = 2\text{-}5$

(22b) $T_n(m) = 1/6\ \tau_2(m) \qquad m = 1,2; \quad n = 9\text{-}11, 14\text{-}16$

(22c) $T_n(m) = 1/8\ \tau_3(m) \qquad m = 1,2; \quad n = 20\text{-}23, 26\text{-}29$

(22d) $T_n(m) = 1/8\ \tau_4(m) \qquad m = 1,2; \quad n = 38\text{-}41, 44\text{-}47$

(22e) $T_n(m) = 1/8\ \tau_5(m) \qquad m = 1,2; \quad n = 50\text{-}53, 56\text{-}58$

(22f) $T_n(m) = 0 \quad m = 1,2; \ n = 1, 6\text{-}8, 12\text{-}13, 17\text{-}19, 24\text{-}25, 30\text{-}37, 42\text{-}43, 48\text{-}49, 54\text{-}55, 60\text{-}72$

With these definitions, the projection equations (12), (13a) and (15a) can be rewritten as follows:

$P_n(2) = 0$ $\quad n = 1,7,8,19,31,32$

$P_n(2) = 1/4\ \tau_1(2)$ $\quad n = 2$

$P_n(2) = U_n\left[CK1_n + 1/4\ \tau_1(1)\right] + 1/4\ \tau_1(2)$ $\quad n = 3\text{-}5$

$P_n(2) = U_n\left[CK1_n + 1/4\ \tau_1(1)\right]$ $\quad n = 6$

$P_n(2) = U_n CK1_n + V_n\left[CK2_n + 1/4\ \tau_1(1)\right] + 1/6\ \tau_2(2)$ $\quad n = 9$

$P_n(2) = U_n\left[CK1_n + 1/6\ \tau_2(1)\right] + V_n\left[CK2_n + 1/4\ \tau_1(1)\right] + 1/6\ \tau_2(2)$ $\quad n = 10,11$

$P_n(2) = U_n\left[CK1_n + 1/6\ \tau_2(1)\right] + V_n\left[CK2_n + 1/4\ \tau_1(1)\right]$ $\quad n = 12$

$P_n(2) = U_n CK1_n + 1/6\ \tau_2(2)$ $\quad n = 14$

$P_n(2) = U_n\left[CK1_n + 1/6\ \tau_2(1)\right] + 1/6\ \tau_2(2)$ $\quad n = 15,16$

$P_n(2) = U_n\left[CK1_n + 1/6\ \tau_2(1)\right]$ $\quad n = 17$

$P_n(2) = U_n CK1_n$ $\quad n = 13,18,43,61\text{-}72$

$P_n(2) = 1/8\ \tau_3(2)$ $\quad n = 20$

$P_n(2) = U_n\left[CK1_n + 1/8\ \tau_3(1)\right] + W_n CK3_n + 1/8\ \tau_3(2)$ $\quad n = 21$

$P_n(2) = U_n\left[CK1_n + 1/8\ \tau_3(1)\right] + W_n\left[CK3_n + 1/6\ \tau_2(1)\right] + 1/8\ \tau_3(2)$ $\quad n = 22,23,27\text{-}29$

$P_n(2) = U_n\left[CK1_n + 1/8\ \tau_3(1)\right] + W_n\left[CK3_n + 1/6\ \tau_2(1)\right]$ $\quad n = 24$

$P_n(2) = U_n CK1_n + W_n CK3_n$ $\quad n = 25,37,55$

$P_n(2) = U_n CK1_n + W_n CK3_n + 1/8\ \tau_3(2)$ $\quad n = 26$

$P_n(2) = U_n\left[CK1_n + 1/8\ \tau_3(1)\right] + W_n CK3_n$ $\quad n = 30$

$P_n(2) = U_n CK1_n + W_n\left[CK3_n + 1/8\ \tau_3(1)\right]$ $\quad n = 33\text{-}36$

$P_n(2) = U_n CK1_n + W_n CK3_n + 1/8\ \tau_4(2)$ $\quad n = 38$

$$P_n(2) = U_n\left[CK1_n + 1/8\ \tau_4(1)\right] + W_n\left[CK3_n + 1/8\ \tau_3(1)\right] + 1/8\ \tau_4(2) \qquad n = 39\text{-}41$$

$$P_n(2) = U_n\left[CK1_n + 1/8\ \tau_4(1)\right] + W_n\left[CK3_n + 1/8\ \tau_3(1)\right] \qquad n = 42$$

$$P_n(2) = U_n CK1_n + 1/8\ \tau_4(2) \qquad n = 44$$

$$P_n(2) = U_n\left[CK1_n + 1/8\ \tau_4(1)\right] + 1/8\ \tau_4(2) \qquad n = 45\text{-}47$$

$$P_n(2) = U_n\left[CK1_n + 1/8\ \tau_4(1)\right] \qquad n = 48$$

$$P_n(2) = W_n CK3_n \qquad n = 49$$

$$P_n(2) = U_n D_{n-1}(0) + W_n CK3_n + 1/8\ \tau_5(2) \qquad n = 50$$

$$P_n(2) = U_n\left[CK1_n + 1/8\ \tau_5(1)\right] + W_n\left[CK3_n + 1/8\ \tau_4(1)\right] + 1/8\ \tau_5(2) \qquad n = 51\text{-}53,\ 57\text{-}59$$

$$P_n(2) = U_n\left[CK1_n + 1/8\ \tau_5(1)\right] + W_n\left[CK3_n + 1/8\ \tau_4(1)\right] \qquad n = 54, 60$$

$$P_n(2) = U_n CK1_n + W_n CK3_n + 1/8\ \tau_5(2) \qquad n = 56$$

With an equation now available for each cell of the inventory matrix at the end of the two years of projection, it is now possible to generate the desired restraint equations. Because of the manner in which requirements are stated the remaining five restraint equations are more easily derived in reverse order.

Recall the tenth requirement equation is

$$\text{(20j)} \qquad \sum_{n=49}^{72} P_n(2) \geq R_5(2)$$

and by proper substitution from equation (23), the tenth restraint equation is

$$\text{(24)} \qquad 1/8\left[\sum_{n=51}^{54} W_n + \sum_{n=57}^{60} W_n\right]\tau_4(1) + 1/8\left[\sum_{n=51}^{54} U_n + \sum_{n=57}^{60} U_n\right]\tau_5(1) + \tau_5(2)$$

$$\geq R_5(2) - \left[\sum_{n=51}^{72} U_n CK1_n + \sum_{n=49}^{60} W_n CK3_n + U_{50} D_{49}(0)\right]$$

where the appropriate elements (a_{ij}) of the matrix of coefficients, A, and the elements (B_i) of the restraint vector B are defined as follows:

$$a_{11,4} = 1/8\left[\sum_{n=51}^{54} W_n + \sum_{n=57}^{60} W_n\right]$$

$$a_{11,5} = 1/8\left[\sum_{n=51}^{54} U_n + \sum_{n=57}^{60} U_n\right]$$

$$B_{11} = R_5(2) - \left[\sum_{n=51}^{72} U_n CK1_n + \sum_{n=49}^{60} W_n CK3_n + U_{50} D_{49}(0)\right] = R_5(2) - S_{10}$$

Therefore we write equation (24) as

$$(24a) \qquad a_{11,4}\,\tau_4(1) + a_{11,5}\,\tau_5(1) + \tau_5(2) \geq B_{11}$$

Recall the ninth requirement equation is

$$(20i) \qquad \sum_{n=31}^{72} P_n(2) = \sum_{n=31}^{48} P_n(2) + \sum_{n=49}^{72} P_n(2) \geq R_4(2)$$

and by proper substitution from equations (23) and (24a) the ninth restraint equation is

$$(25) \qquad 1/8\left[\sum_{n=33}^{36} W_n + \sum_{n=39}^{42} W_n\right]\tau_3(1) + 1/8\left[\sum_{n=39}^{42} U_n + \sum_{n=45}^{48} U_n\right]\tau_4(1) + a_{11,4}\tau_4(1)$$

$$+ a_{11,5}\,\tau_5(1) + \tau_4(2) + \tau_5(2) \geq R_4(2) - \left[S_{10} + \sum_{n=33}^{48} U_n CK1_n + \sum_{n=33}^{42} W_n CK3_n\right]$$

Defining (a_{ij}) and (B_i) similarly we have

$$a_{10,3} = 1/8\left[\sum_{n=33}^{36} W_n + \sum_{n=39}^{42} W_n\right]$$

$$a_{10,4} = 1/8\left[\sum_{n=39}^{42} U_n + \sum_{n=45}^{48} U_n\right] + a_{11,4}$$

$$B_{10} = R_4(2) - \left[S_{10} + \sum_{n=33}^{48} U_n CK1_n + \sum_{n=33}^{42} W_n CK3_n\right] = R_4(2) - \left[S_{10} + S_9\right]$$

Therefore we rewrite equation (25) as

$$\text{(25a)} \qquad a_{10,3}\ \tau_3(1) + a_{10,4}\ \tau_4(1) + a_{11,5}\ \tau_5(1) + \tau_4(2) + \tau_5(2) \geq B_{10}$$

Recall the eighth requirement equation is

$$\text{(20h)} \qquad \sum_{n=19}^{72} P_n(2) = \sum_{n=19}^{30} P_n(2) + \sum_{n=31}^{72} P_n(2) \geq R_3(2)$$

and by proper substitution from equations (23) and (25a) the eighth restraint equation becomes

$$\text{(26)} \qquad 1/6\left[\sum_{n=22}^{24} W_n + \sum_{n=27}^{29} W_n\right] \tau_2(1) + 1/8\left[\sum_{n=21}^{24} U_n + \sum_{n=27}^{30} U_n\right]\tau_3(1) + a_{10,3}\ \tau_3(1)$$

$$+ a_{10,4}\ \tau_4(1) + a_{11,5}\ \tau_5(1) + \tau_3(2) + \tau_4(2) + \tau_5(2)$$

$$\geq R_3(2) - \left[S_{10} + S_9 + \sum_{n=21}^{30}\left(U_n CK1_n + W_n CK3_n\right)\right]$$

Defining (a_{ij}) and (B_i) similarly, we have

$$a_{9,2} = 1/6\left[\sum_{n=22}^{24} W_n + \sum_{n=27}^{29} W_n\right]$$

$$a_{9,3} = 1/8\left[\sum_{n=21}^{24} U_n + \sum_{n=27}^{30} U_n\right] + a_{10,3}$$

$$B_9 = R_3(2) - \left[S_{10} + S_9 + \sum_{n=21}^{30}\left(U_n CK1_n + W_n CK3_n\right)\right] = R_3(2) - \left[S_{10} + S_9 + S_8\right]$$

Therefore we rewrite equation (26) as

$$(26a)\qquad a_{9,2}\,\tau_2(1) + a_{9,3}\,\tau_3(1) + a_{10,4}\,\tau_4(1) + a_{11,5}\,\tau_5(1) + \tau_3(2) + \tau_4(2) + \tau_5(2) \geq B_9$$

Recall that the seventh requirement equation is

$$(20g)\qquad \sum_{n=7}^{72} P_n(2) = \sum_{n=7}^{18} P_n(2) + \sum_{n=19}^{72} P_n(2) \geq R_2(2)$$

and by proper substitution from equations (23) and (26a), the seventh restraint equation becomes

$$(27)\qquad 1/4\left[\sum_{n=9}^{12} V_n\right]\tau_1(1) + 1/6\left[\sum_{n=10}^{12} U_n + \sum_{n=15}^{17} U_n\right]\tau_2(1) + a_{9,2}\,\tau_2(1) + a_{9,3}\,\tau_3(1)$$

$$+ a_{10,4}\,\tau_4(1) + a_{11,5}\,\tau_5(1) + \tau_2(2) + \tau_3(2) + \tau_4(2) + \tau_5(2)$$

$$\geq R_2(2) - \left[S_{10} + S_9 + S_8 + \sum_{n=9}^{18} U_n CK1_n + \sum_{n=9}^{12} V_n CK2_n\right]$$

Defining (a_{ij}) and (B_i) similarly, we have

$$a_{8,1} = 1/4\left[\sum_{n=9}^{12} V_n\right]$$

$$a_{8,2} = 1/6\left[\sum_{n=10}^{12} U_n + \sum_{n=15}^{17} U_n\right] + a_{9,2}$$

$$B_8 = R_2(2) - \left[S_{10} + S_9 + S_8 + \sum_{n=9}^{18} U_n CK1_n + \sum_{n=9}^{12} V_n CK2_n\right] =$$

$$= R_2(2) - \left[S_{10} + S_9 + S_8 + S_7\right]$$

Therefore we rewrite equation (27) as

(27a) $$a_{8,1}\tau_1(1) + a_{8,2}\tau_2(1) + a_{9,3}\tau_3(1) + a_{10,4}\tau_4(1) + a_{11,5}\tau_5(1) + $$
$$+ \tau_2(2) + \tau_3(2) + \tau_4(2) + \tau_5(2) \geq B_8$$

Recall the sixth requirement equation is

(20f) $$\sum_{n=1}^{72} P_n(2) = \sum_{n=1}^{6} P_n(2) + \sum_{n=7}^{72} P_n(2) \geq R_1(2)$$

and by proper substitution from equations (23) and (27a) the sixth restraint equation equation becomes

(28) $$1/4\left[\sum_{n=3}^{7} U_n\right]\tau_1(1) + a_{8,1}\tau_1(1) + a_{8,2}\tau_2(1) + a_{9,3}\tau_3(1) + a_{10,4}\tau_4(1)$$
$$+ a_{11,5}\tau_5(1) + \tau_1(2) + \tau_2(2) + \tau_3(2) + \tau_4(2) + \tau_5(2) \geq$$
$$\geq R_1(2) - \left[S_{10} + S_9 + S_8 + S_7 + \sum_{n=3}^{6} U_n CK1_n\right]$$

Defining (a_{ij}) and (B_i) similarly we have

$$a_{7,1} = 1/4\left[\sum_{n=3}^{7} U_n\right] + a_{8,1}$$

$$B_7 = R_1(2) - \left[S_{10} + S_9 + S_8 + S_7 + \sum_{n=3}^{6} U_n CK1_n\right] = R_1(2) - \left[S_{10} + S_9 + S_8 + S_7 + S_6\right]$$

Therefore we rewrite equation (28) as

(28a) $$a_{7,1}\tau_1(1) + a_{8,2}\tau_2(1) + a_{9,3}\tau_3(1) + a_{10,4}\tau_4(1) + a_{11,5}\tau_5(1) +$$
$$+ \tau_1(2) + \tau_2(2) + \tau_3(2) + \tau_4(2) + \tau_5(2) \geq B_7$$

Thus the five restraint equations generated from the second year requirements are equations (24a), (25a), (26a), (27a), and (28a). These equations, in addition to the five equations (11a) through (11e), provide the ten linear restraint equations for the two year model. They serve to provide the inputs (A, B, cY) to the linear program which will minimize the training input during both years of the two year projection, while meeting requirements both years.

These inputs are:

$$A = \left[a_{ij} \right] = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ a_{7,1} & a_{8,2} & a_{9,3} & a_{10,4} & a_{11,5} & 1 & 1 & 1 & 1 & 1 \\ a_{8,1} & a_{8,2} & a_{9,3} & a_{10,4} & a_{11,5} & 0 & 1 & 1 & 1 & 1 \\ 0 & a_{9,2} & a_{9,3} & a_{10,4} & a_{11,5} & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & a_{10,3} & a_{10,4} & a_{11,5} & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & a_{11,4} & a_{11,5} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Note: The rows of A ($i = 1, 2, \ldots, 10$) represent, in order, the ten restraint equations (11a-11e, 24a, 25a, 26a, 27a, 28a). The elements of each row are the coefficients of the ten unknown training inputs $\tau_k(m)$ ($k = 1, 2, \ldots, 5$; $m = 1, 2$).

$$B = \left[B_i \right] = \begin{bmatrix} B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \\ B_7 \\ B_8 \\ B_9 \\ B_{10} \\ B_{11} \end{bmatrix}$$

where B_i are defined by equations found in the derivations of the restraint equations;

$$c = \left[c_k \right] = (c_1, c_2, c_3, c_4, c_5)$$

where the c_k are defined in Chapter II;

$$Y = \left[\tau_k(m) \right] \qquad m = 1,2; \; k = 1,2,\ldots,5$$

where the $\tau_k(m)$ represent the ten training inputs during the two years, and the objective function is

$$\text{minimize } cY = \min \sum_{m=1}^{2} \sum_{k=1}^{5} c_k \, \tau_k(m)$$

With the ten training inputs having been determined by the linear program, equations (22a) through (22f) are used to determine the training inputs to each of the 72 cells during both years of projection, $T_n(m)$, ($m = 1,2$; $n = 1,2,\ldots,72$). Knowing the values of all $T_n(m)$, the

projection equation (23) is then used to determine each $P_n(2)$, $(n = 1, 2, \ldots, 72)$, and hence the total projected inventory after two years, $P(2)$.

b. Five Year Model

This section of Appendix B lists the explicit expressions for the inputs to the linear program.

1. Matrix of Coefficients, $A = (a_{ij})$

The matrix of coefficients, $A = (a_{ij})$, for the five year model is shown in Figure B-1.

The computational formulae for describing the matrix $A = (a_{ij})$ for the five year model are as follows:

$$a_{7,1} = 1/4\left[\sum_{n=3}^{6} U_n\right] + a_{8,1}$$

$$a_{8,1} = 1/4\left[\sum_{n=9}^{12} V_n\right]$$

$$a_{8,2} = 1/6\left[\sum_{n=10}^{12} U_n + \sum_{n=15}^{17} U_n\right] + a_{9,2}$$

$$a_{9,2} = 1/6\left[\sum_{n=22}^{24} W_n + \sum_{n=27}^{29} W_n\right]$$

$$a_{9,3} = 1/8\left[\sum_{n=21}^{24} U_n + \sum_{n=27}^{30} U_n\right] + a_{10,3}$$

$$a_{10,3} = 1/8\left[\sum_{n=33}^{36} W_n + \sum_{n=39}^{42} W_n\right]$$

$$a_{10,4} = 1/8\left[\sum_{n=39}^{42} U_n + \sum_{n=45}^{48} U_n\right] + a_{11,4}$$

$$a_{11,4} = 1/8\left[\sum_{n=51}^{54} W_n + \sum_{n=57}^{60} W_n\right]$$

$$a_{12,1} = 1/3\left[\sum_{n=4}^{6} U_n\right]\left[a_{7,1} - a_{8,1}\right] + a_{13,1}$$

$$a_{13,1} = 1/4\left[\sum_{n=10}^{13} U_n\right]a_{8,1} + 1/4\left[\sum_{n=10}^{13} V_n\right]\left[a_{7,1} - a_{8,1}\right] + a_{14,1}$$

$$a_{13,2} = 1/6\left[\sum_{n=11}^{13} U_n + \sum_{n=16}^{18} U_n\right]\left[a_{8,2} - a_{9,2}\right] + a_{14,2}$$

	c_1	c_2	c_3	c_4	c_5	c_1	c_2	c_3	c_4	c_5	c_1	c_2	c_3	c_4	c_5	c_1	c_2	c_3	c_4	c_5	c_1	c_2	c_3	c_4	c_5
First Year	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Second Year	$a_{7,1}$	$a_{8,2}$	$a_{9,3}$	$a_{10,4}$	$a_{11,5}$	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$a_{8,1}$	$a_{8,2}$	$a_{9,3}$	$a_{10,4}$	$a_{11,5}$	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	$a_{9,2}$	$a_{9,3}$	$a_{10,4}$	$a_{11,5}$	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	$a_{10,3}$	$a_{10,4}$	$a_{11,5}$	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	$a_{11,4}$	$a_{11,5}$	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Third Year	$a_{12,1}$	$a_{13,2}$	$a_{14,3}$	$a_{15,4}$	$a_{16,5}$	$a_{7,1}$	$a_{8,2}$	$a_{9,3}$	$a_{10,4}$	$a_{11,5}$	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
	$a_{13,1}$	$a_{13,2}$	$a_{14,3}$	$a_{15,4}$	$a_{16,5}$	$a_{8,1}$	$a_{8,2}$	$a_{9,3}$	$a_{10,4}$	$a_{11,5}$	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0
	$a_{14,1}$	$a_{14,2}$	$a_{14,3}$	$a_{15,4}$	$a_{16,5}$	0	$a_{9,2}$	$a_{9,3}$	$a_{10,4}$	$a_{11,5}$	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
	0	$a_{15,2}$	$a_{15,3}$	$a_{15,4}$	$a_{16,5}$	0	0	$a_{10,3}$	$a_{10,4}$	$a_{11,5}$	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
	0	0	$a_{16,3}$	$a_{16,4}$	$a_{16,5}$	0	0	0	$a_{11,4}$	$a_{11,5}$	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
Fourth Year	$a_{17,1}$	$a_{18,2}$	$a_{19,3}$	$a_{20,4}$	$a_{21,5}$	$a_{12,1}$	$a_{13,2}$	$a_{14,3}$	$a_{15,4}$	$a_{16,5}$	$a_{7,1}$	$a_{8,2}$	$a_{9,3}$	$a_{10,4}$	$a_{11,5}$	1	1	1	1	1	0	0	0	0	0
	$a_{18,1}$	$a_{18,2}$	$a_{19,3}$	$a_{20,4}$	$a_{21,5}$	$a_{13,1}$	$a_{13,2}$	$a_{14,3}$	$a_{15,4}$	$a_{16,5}$	$a_{8,1}$	$a_{8,2}$	$a_{9,3}$	$a_{10,4}$	$a_{11,5}$	0	1	1	1	1	0	0	0	0	0
	$a_{19,1}$	$a_{19,2}$	$a_{19,3}$	$a_{20,4}$	$a_{21,5}$	$a_{14,1}$	$a_{14,2}$	$a_{14,3}$	$a_{15,4}$	$a_{16,5}$	0	$a_{9,2}$	$a_{9,3}$	$a_{10,4}$	$a_{11,5}$	0	0	1	1	1	0	0	0	0	0
	$a_{20,1}$	$a_{20,2}$	$a_{20,3}$	$a_{20,4}$	$a_{21,5}$	0	$a_{15,2}$	$a_{15,3}$	$a_{15,4}$	$a_{16,5}$	0	0	$a_{10,3}$	$a_{10,4}$	$a_{11,5}$	0	0	0	1	1	0	0	0	0	0
	0	$a_{21,2}$	$a_{21,3}$	$a_{21,4}$	$a_{21,5}$	0	0	$a_{16,3}$	$a_{16,4}$	$a_{16,5}$	0	0	0	$a_{11,4}$	$a_{11,5}$	0	0	0	0	1	0	0	0	0	0
Fifth Year	$a_{22,1}$	$a_{23,2}$	$a_{24,3}$	$a_{25,4}$	$a_{26,5}$	$a_{17,1}$	$a_{18,2}$	$a_{19,3}$	$a_{20,4}$	$a_{21,5}$	$a_{12,1}$	$a_{13,2}$	$a_{14,3}$	$a_{15,4}$	$a_{16,5}$	$a_{7,1}$	$a_{8,2}$	$a_{9,3}$	$a_{10,4}$	$a_{11,5}$	1	1	1	1	1
	$a_{23,1}$	$a_{23,2}$	$a_{24,3}$	$a_{25,4}$	$a_{26,5}$	$a_{18,1}$	$a_{18,2}$	$a_{19,3}$	$a_{20,4}$	$a_{21,5}$	$a_{13,1}$	$a_{13,2}$	$a_{14,3}$	$a_{15,4}$	$a_{16,5}$	$a_{8,1}$	$a_{8,2}$	$a_{9,3}$	$a_{10,4}$	$a_{11,5}$	0	1	1	1	1
	$a_{24,1}$	$a_{24,2}$	$a_{24,3}$	$a_{25,4}$	$a_{26,5}$	$a_{19,1}$	$a_{19,2}$	$a_{19,3}$	$a_{20,4}$	$a_{21,5}$	$a_{14,1}$	$a_{14,2}$	$a_{14,3}$	$a_{15,4}$	$a_{16,5}$	0	$a_{9,2}$	$a_{9,3}$	$a_{10,4}$	$a_{11,5}$	0	0	1	1	1
	$a_{25,1}$	$a_{25,2}$	$a_{25,3}$	$a_{25,4}$	$a_{26,5}$	$a_{20,1}$	$a_{20,2}$	$a_{20,3}$	$a_{20,4}$	$a_{21,5}$	0	$a_{15,2}$	$a_{15,3}$	$a_{15,4}$	$a_{16,5}$	0	0	$a_{10,3}$	$a_{10,4}$	$a_{11,5}$	0	0	0	1	1
	$a_{26,1}$	$a_{26,2}$	$a_{26,3}$	$a_{26,4}$	$a_{26,5}$	0	$a_{21,2}$	$a_{21,3}$	$a_{21,4}$	$a_{21,5}$	0	0	$a_{16,3}$	$a_{16,4}$	$a_{16,5}$	0	0	0	$a_{11,4}$	$a_{11,5}$	0	0	0	0	1

FIGURE B-1

MATRIX OF COEFFICIENTS FOR FIVE-YEAR MODEL

FIGURE 8-1

$$a_{14,1} = 1/4\left[\sum_{n=22}^{25} W_n\right]$$

$$a_{14,2} = 1/6\left[\sum_{n=23}^{25} Y_n + \sum_{n=28}^{30} Y_n\right] a_{9,2} + 1/6\left[\sum_{n=22}^{25} W_n + \sum_{n=28}^{30} W_n\right]\left[a_{8,2} - a_{9,2}\right]$$

$$a_{14,3} = 1/7\left[\sum_{n=22}^{25} U_n + \sum_{n=28}^{30} U_n\right]\left[a_{9,3} - a_{10,3}\right] + a_{15,3}$$

$$a_{15,3} = 1/8\left[\sum_{n=34}^{37} Y_n + \sum_{n=40}^{43} Y_n\right] a_{10,3} + 1/8\left[\sum_{n=34}^{37} W_n + \sum_{n=40}^{43} W_n\right]\left[a_{9,3} - a_{10,3}\right]$$

$$a_{15,4} = 1/7\left[\sum_{n=40}^{43} U_n + \sum_{n=46}^{48} U_n\right]\left[a_{10,4} - a_{11,4}\right] + a_{16,4}$$

$$a_{16,4} = 1/8\left[\sum_{n=52}^{55} U_n + \sum_{n=58}^{61} U_n\right] a_{11,4} + 1/8\left[\sum_{n=52}^{55} W_n + \sum_{n=58}^{61} W_n\right]\left[a_{10,4} - a_{11,4}\right]$$

$$a_{16,5} = 1/8\left[\sum_{n=52}^{55} U_n + \sum_{n=58}^{61} U_n\right] a_{11,5}$$

$$a_{17,1} = 1/2\left[\sum_{n=5}^{6} U_n\right]\left[a_{12,1} - a_{13,1}\right] + a_{18,1}$$

$$a_{18,1} = 1/3\left[\sum_{n=11}^{13} U_n\right] a_{13,1} + 1/3\left[\sum_{n=11}^{13} V_n\right]\left[a_{12,1} - a_{13,1}\right] + a_{19,1}$$

$$a_{18,2} = 1/5\left[\sum_{n=12}^{14} U_n + \sum_{n=17}^{18} U_n\right]\left[a_{13,2} - a_{14,2}\right] + a_{19,2}$$

$$a_{19,1} = 1/4\left[\sum_{n=23}^{26} Y_n\right] a_{14,1} + 1/4\left[\sum_{n=23}^{26} W_n\right]\left[a_{13,1} - a_{14,1}\right]$$

$$a_{19,2} = 1/5\left[\sum_{n=24}^{26} U_n + \sum_{n=29}^{30} U_n\right] a_{14,2} + 1/5\left[\sum_{n=23}^{25} W_n + \sum_{n=29}^{30} W_n\right]\left[a_{13,2} - a_{14,2}\right] + a_{20,2}$$

$$a_{19,3} = 1/5\left[\sum_{n=24}^{26} U_n + \sum_{n=29}^{30} U_n\right]\left[a_{14,3} - a_{15,3}\right] + a_{20,3}$$

$$a_{20,2} = 1/6\left[\sum_{n=36}^{38} W_n + \sum_{n=41}^{43} W_n\right]$$

$$a_{20,3} = 1/8\left[\sum_{n=35}^{38} Y_n + \sum_{n=41}^{44} Y_n\right] a_{15,3} + 1/8\left[\sum_{n=34}^{37} W_n + \sum_{n=40}^{43} W_n\right]\left[a_{14,3} - a_{15,3}\right]$$

$$a_{20,4} = 1/6\left[\sum_{n=41}^{44} U_n + \sum_{n=47}^{48} U_n\right]\left[a_{15,4} - a_{16,4}\right] + a_{21,4}$$

$$a_{21,4} = 1/8\left[\sum_{n=53}^{56} U_n + \sum_{n=59}^{62} U_n\right] a_{16,4} + 1/7\left[\sum_{n=53}^{56} W_n + \sum_{n=59}^{61} W_n\right]\left[a_{15,4} - a_{16,4}\right]$$

$$a_{21,5} = 1/8\left[\sum_{n=53}^{56} U_n + \sum_{n=59}^{62} U_n\right] a_{16,5}$$

$$a_{22,1} = U_6\left[a_{17,1} - a_{18,1}\right] + a_{23,1}$$

$$a_{23,1} = 1/2\left[\sum_{n=12}^{13} U_n\right] a_{18,1} + 1/2\left[\sum_{n=12}^{13} V_n\right]\left[a_{17,1} - a_{18,1}\right] + a_{24,1}$$

$$a_{23,2} = 1/4\left[\sum_{n=13}^{15} U_n + U_{18}\right]\left[a_{18,2} - a_{19,2}\right] + a_{24,2}$$

$$a_{24,1} = 1/4\left[\sum_{n=24}^{27} U_n\right] a_{19,1} + 1/4\left[\sum_{n=24}^{27} W_n\right]\left[a_{18,1} - a_{19,1}\right] + a_{25,1}$$

$$a_{24,2} = 1/4\left[\sum_{n=25}^{27} U_n + U_{30}\right] a_{19,2} + 1/4\left[\sum_{n=25}^{27} W_n + W_{30}\right]\left[a_{18,2} - a_{19,2}\right] + a_{25,2}$$

$$a_{24,3} = 1/4\left[\sum_{n=25}^{27} U_n + U_{30}\right]\left[a_{19,3} - a_{20,3}\right] + a_{25,3}$$

$$a_{25,1} = 1/4\left[\sum_{n=36}^{39} W_n\right] a_{19,1}$$

$$a_{25,2} = 1/5\left[\sum_{n=37}^{39} Y_n + \sum_{n=47}^{48} Y_n\right] a_{20,2} + 1/6\left[\sum_{n=37}^{39} W_n + \sum_{n=47}^{49} W_n\right]\left[a_{19,2} - a_{20,2}\right]$$

$$a_{25,3} = 1/5\left[\sum_{n=42}^{45} U_n + U_{48}\right] a_{20,3} + 1/7\left[\sum_{n=35}^{38} W_n + \sum_{n=41}^{43} W_n\right]\left[a_{19,3} - a_{20,3}\right] + a_{26,3}$$

$$a_{25,4} = 1/5\left[\sum_{n=42}^{45} U_n + U_{48}\right]\left[a_{20,4} - a_{21,4}\right] + a_{26,4}$$

$$a_{26,3} = 1/3\left[\sum_{n=49}^{51} W_n\right] a_{20,3}$$

$$a_{26,4} = 1/8\left[\sum_{n=54}^{57} U_n + \sum_{n=60}^{63} U_n\right] a_{21,4} + 1/6\left[\sum_{n=54}^{57} W_n + \sum_{n=60}^{61} W_n\right]\left[a_{20,4} - a_{21,4}\right]$$

$$a_{26,5} = 1/8\left[\sum_{n=54}^{57} U_n + \sum_{n=60}^{63} U_n\right] a_{21,5}$$

2. Restraint Vector, $B = (B_i)$

The restraint vector for the five year model has 25 elements, five for each year. The elements are computed for each year by a method similar to that used in the one and two year models.

$$B = \begin{bmatrix} B_2 \\ \vdots \\ B_6 \\ B_7 \\ \vdots \\ B_{11} \\ B_{12} \\ \vdots \\ B_{16} \\ B_{17} \\ \vdots \\ B_{21} \\ B_{22} \\ \vdots \\ B_{26} \end{bmatrix} \begin{matrix} \left.\vphantom{\begin{matrix} B_2 \\ \vdots \\ B_6 \end{matrix}}\right\} \text{First Year} \\ \left.\vphantom{\begin{matrix} B_7 \\ \vdots \\ B_{11} \end{matrix}}\right\} \text{Second Year} \\ \left.\vphantom{\begin{matrix} B_{12} \\ \vdots \\ B_{16} \end{matrix}}\right\} \text{Third Year} \\ \left.\vphantom{\begin{matrix} B_{17} \\ \vdots \\ B_{21} \end{matrix}}\right\} \text{Fourth Year} \\ \left.\vphantom{\begin{matrix} B_{22} \\ \vdots \\ B_{26} \end{matrix}}\right\} \text{Fifth Year} \end{matrix}$$

3 Cost Vector, $c = \left[c_k \right]$

The cost vector is defined in Chapter II.

4. Unknown Training Input, Y

$Y = \left[\tau_k(m) \right]$, $m = 1,2,\ldots,5;\ k = 1,2,\ldots,5;$

where $\tau_k(m)$ represents the 25 training inputs during the five years.

5. Objective Function, min cY

$$\min cY = \min \sum_{m=1}^{5} \sum_{k=1}^{5} c_k \tau_k(m)$$

APPENDIX C

SENSITIVITY ANALYSIS

A computer model like PIPE is as effective and accurate as the input parameters that it uses. The accuracy required of the input parameters, however, may be dictated by the effect that changes in these input parameters have on the resultant solution. Sensitivity analysis is the method by which this effect is evaluated.

PIPE FIVE was evaluated for sensitivity by successively operating the model with all parameters at "standard" values, (as explained in Section II C), except one. This one was, in turn, attrition rate, promotion rate, reenlistment rate, cost coefficients, and control factor. The effects of varying these parameters, one at a time, is detailed in the following sections.

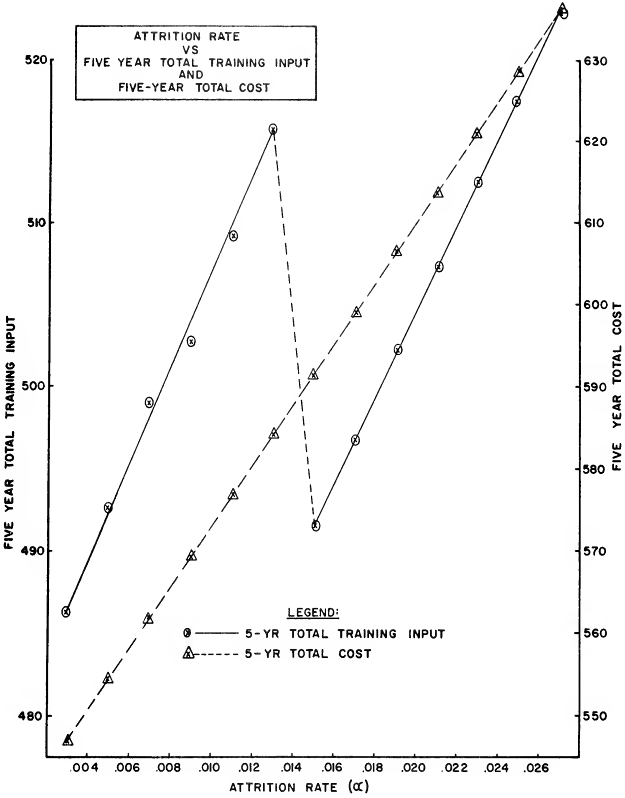
1. Variation of Attrition Rate

The five-year model was operated with all parameters at standard values except attrition rate, which was varied from 0.003 to 0.027 in increments of 0.002. Figure C-1 illustrates the results of these projections, the following of which are significant:

a. Total five-year cost increases linearly as attrition rate increases. This is intuitively the expected result, since the program should have higher cost if more men are leaving the program through attrition.

b. The curve for the total five-year input has a sharp break in the interval where the attrition rate is 0.013 to 0.015. This demonstrates the fact that for the minimum cost solution to the problem, the inputs of personnel consist of men in the lower (less expensive) pay grades. Near an attrition of 0.015, however, it becomes less expensive to put in <u>less</u> men in the <u>higher</u> pay grades, (see Figure C-2A). This "breakpoint" is not intuitively obvious to the personnel planner, but is clearly delineated by PIPE.

c. Attrition rate has an effect on the number of men required by the program during any given year of projection. Figure C-2B demonstrates that for attrition rates of 0.003 to 0.013, about twice as many men are put into the program during the third year as in the second and fourth years. For attrition rates of 0.015 to 0.027, however, the fourth year of projection requires double the input of men compared to the second and third years.



ATTRITION RATE
VS
FIVE YEAR TOTAL TRAINING INPUT
AND
FIVE-YEAR TOTAL COST
520
510
500
490
480
FIVE YEAR TOTAL TRAINING INPUT
630
620
610
600
590
580
570
560
550
FIVE YEAR TOTAL COST
LEGEND:
5-YR TOTAL TRAINING INPUT
5-YR TOTAL COST
.004 .006 .008 .010 .012 .014 .016 .018 .020 .022 .024 .026
ATTRITION RATE (α)

FIGURE C-1

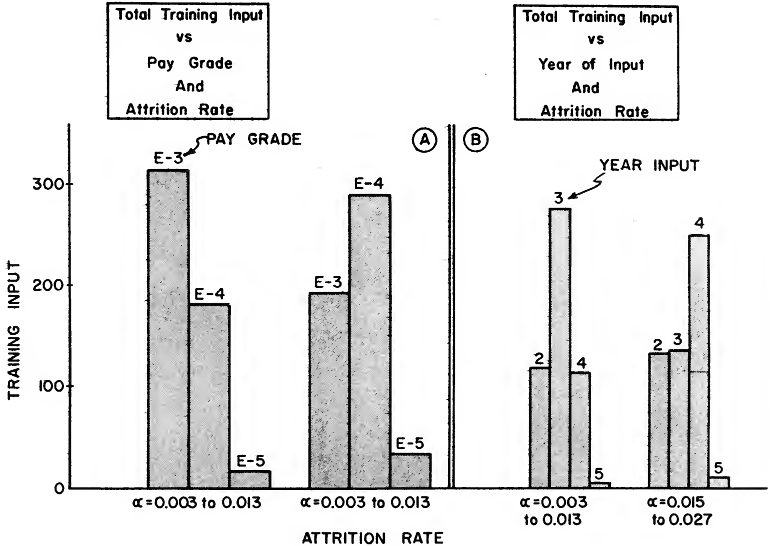

Total Training Input
vs
Pay Grade
And
Attrition Rate
Total Training Input
vs
Year of Input
And
Attrition Rate
PAY GRADE
E-3
E-4
E-5
A
B
YEAR INPUT
2
3
4
5
TRAINING INPUT
300
200
100
0
α=0.003 to 0.013
α=0.003 to 0.013
α=0.003 to 0.013
α=0.015 to 0.027
ATTRITION RATE

FIGURE C-2

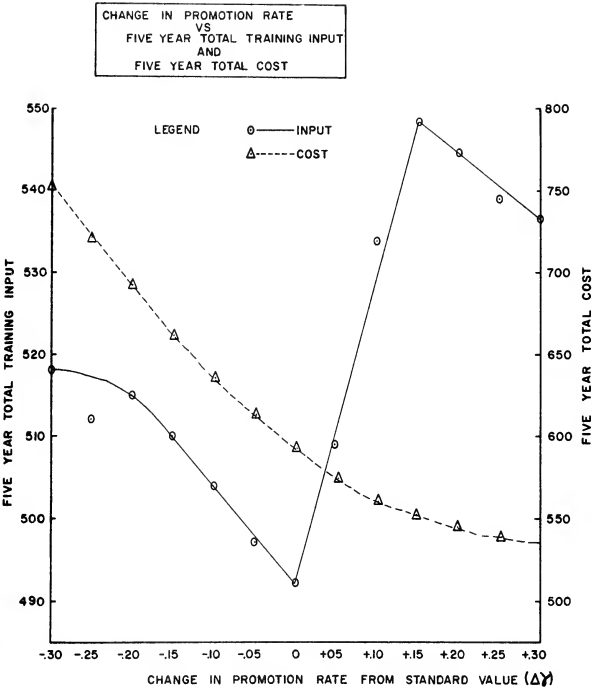
2. Variation of Promotion Rate

a. Figure C-3 illustrates that as the promotion rate is increased for all pay grades, the total cost of meeting requirements decreases while the total training input fluctuates. The reasons for this fluctuation are illustrated in Figure C-4.

b. Figure C-4 illustrates that as the promotion rate increases, the training inputs to E-4, E-5, and E-6 decrease while that of E-3 increases. This happens because the higher promotion rate enables the requirements for the higher pay grade to be met by the increasing portion of the E-3 input that is promoted during the year of projection.

c. The training input solution for pay grade E-7 is zero for all promotion rates considered.

d. It appears from Figure C-3 that the promotion scheme representing the minimum total training input is about at the values of promotion rate used as "standard", (i. e., at delta gamma equal to zero). Thus it may be the case that, for minimum input of personnel over the five-year period, promotion rate should be at the "standard" value, but for minimum cost, promotion rate should be as high as possible.



CHANGE IN PROMOTION RATE
VS
FIVE YEAR TOTAL TRAINING INPUT
AND
FIVE YEAR TOTAL COST
LEGEND
INPUT
COST
FIVE YEAR TOTAL TRAINING INPUT
FIVE YEAR TOTAL COST
550
540
530
520
510
500
490
800
750
700
650
600
550
500
-.30 -.25 -.20 -.15 -.10 -.05 0 +05 +.10 +.15 +.20 +.25 +.30
CHANGE IN PROMOTION RATE FROM STANDARD VALUE (ΔY)

FIGURE C-3

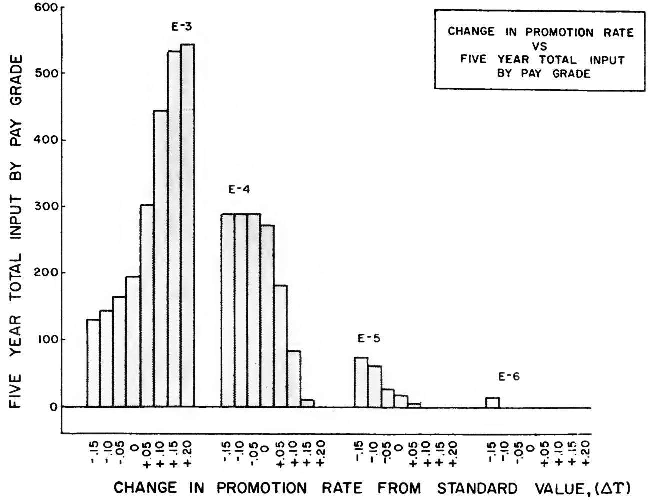

CHANGE IN PROMOTION RATE
VS
FIVE YEAR TOTAL INPUT
BY PAY GRADE
FIVE YEAR TOTAL INPUT BY PAY GRADE
0
100
200
300
400
500
600
E-3
E-4
E-5
E-6
-.15 -.10 -.05 0 +.05 +.10 +.15 +.20
CHANGE IN PROMOTION RATE FROM STANDARD VALUE, (ΔT)

FIGURE C-4

3. Variation of Reenlistment Rate

Sensitivity of the model to changes in reenlistment rate at the end of the first enlistment was performed on all pay grades since it was felt that this was the most critical time in the reenlistment problem. Reenlistment rate was varied in increments of 0.05 to values of -0.30 and +0.30 from the standard value. In the interval from the standard value to +0.12, a fluctuation in the total training input was noted. Further analysis was therefore made of this interval, with reenlistment rate changed by increments of 0.005 in this region.

a. Figure C-5 illustrates that as reenlistment rate is increased for all pay grades, the total cost of meeting requirements decreases while the total training input fluctuates. The reasons for these fluctuations are illustrated in Figure C-6.

b. Figure C-6 illustrates that as the reenlistment rate increases, the training inputs to pay grades E-4 and E-5 decrease while that of E-3 increases. This happens because the higher reenlistment rate enables the requirements for E-4 and E-5 to be met by the increasing portion of the E-3 input that is retained during the years of projection.

c. The training input solution for pay grades E-6 and E-7 are zero for all reenlistment rates considered.

d. Figure C-5 illustrates that the minimum cost solution and the minimum training input solution are both found at the point of maximum reenlistment rate.

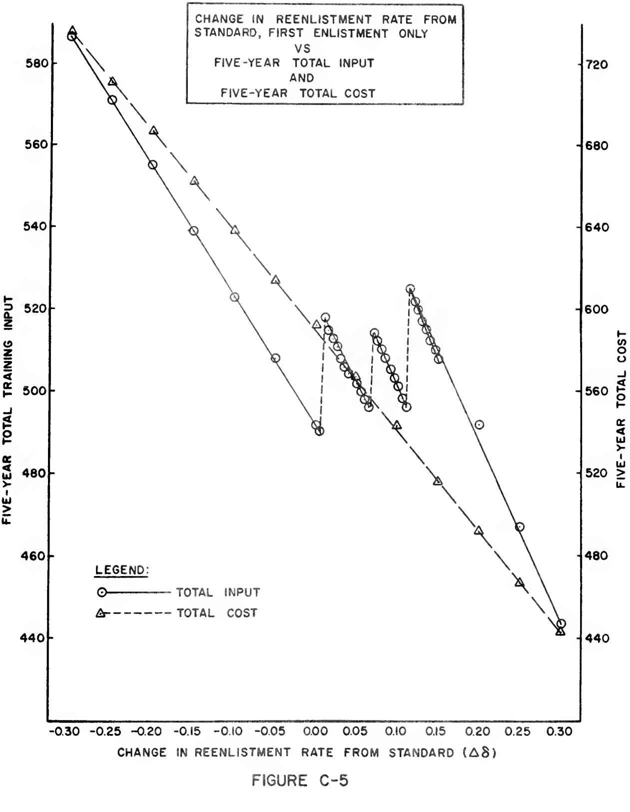
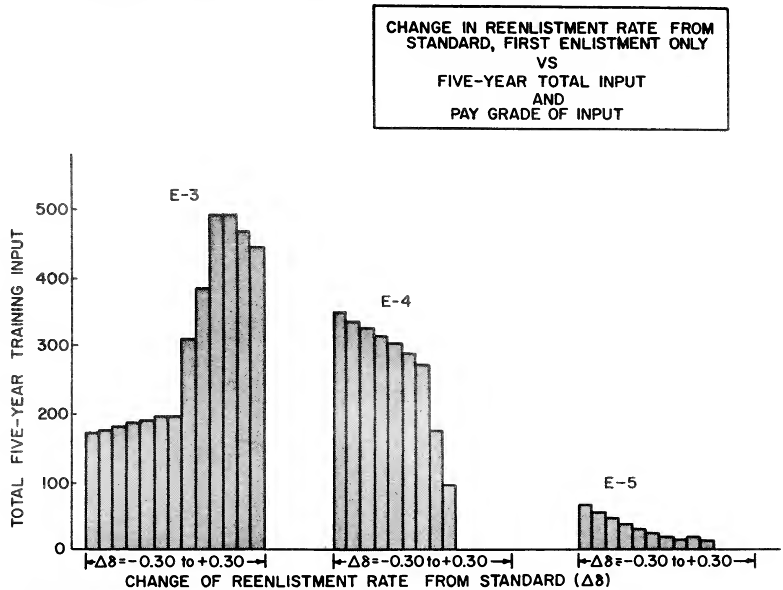

CHANGE IN REENLISTMENT RATE FROM STANDARD, FIRST ENLISTMENT ONLY
VS
FIVE-YEAR TOTAL INPUT
AND
FIVE-YEAR TOTAL COST
FIVE-YEAR TOTAL TRAINING INPUT
FIVE-YEAR TOTAL COST
580
560
540
520
500
480
460
440
720
680
640
600
560
520
480
440
LEGEND:
TOTAL INPUT
TOTAL COST
-0.30 -0.25 -0.20 -0.15 -0.10 -0.05 0.00 0.05 0.10 0.15 0.20 0.25 0.30
CHANGE IN REENLISTMENT RATE FROM STANDARD (Δδ)

FIGURE C-5

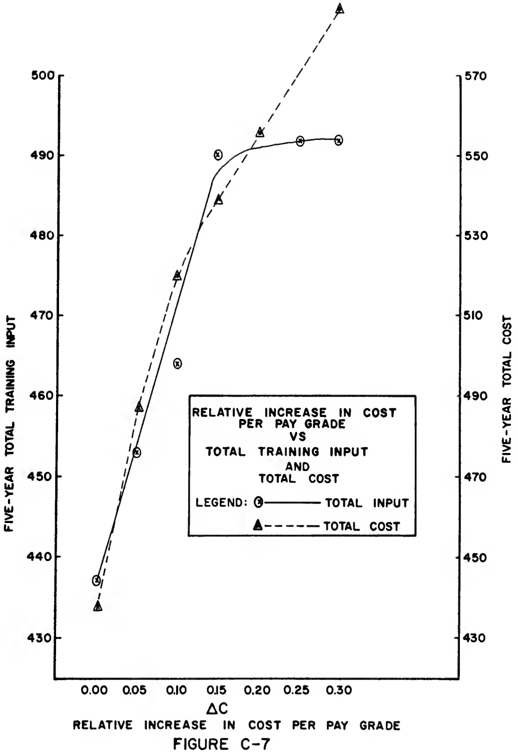

CHANGE IN REENLISTMENT RATE FROM STANDARD, FIRST ENLISTMENT ONLY
VS
FIVE-YEAR TOTAL INPUT
AND
PAY GRADE OF INPUT
TOTAL FIVE-YEAR TRAINING INPUT
0
100
200
300
400
500
E-3
E-4
E-5
Δδ = -0.30 to +0.30
Δδ = -0.30 to +0.30
Δδ = -0.30 to +0.30
CHANGE OF REENLISTMENT RATE FROM STANDARD (Δδ)

4. Cost Coefficients Changed

The cost coefficients in PIPE FIVE represent the relative costs between the different pay grades considered. These cost coefficients were varied by changing the relative cost or value of one rate as compared to another. Thus an increase in relative costs indicates that one rate has become comparatively more expensive to use when compared to a lower rate than had previously been the case.

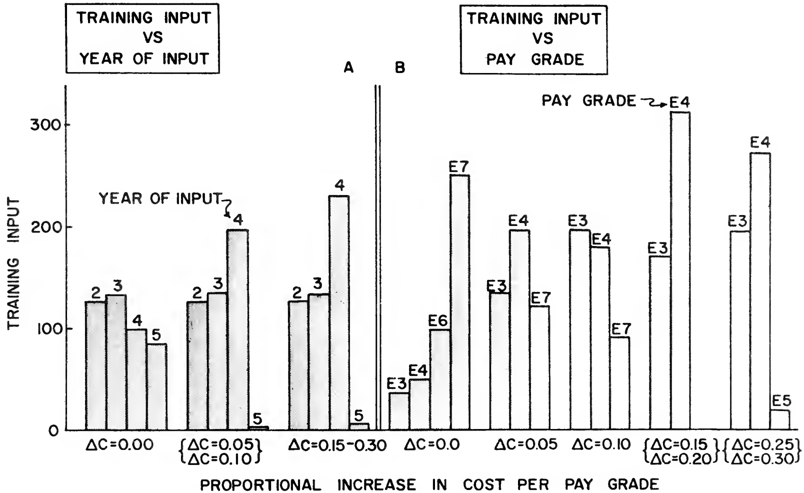
a. The total five-year cost of the program increases almost linearly as relative cost increases, (Figure C-7).

b. Figure C-7 shows that the total five-year training input increases linearly with increase in relative costs to a certain point, (about 0.15), when a plateau is reached. Beyond a relative cost within pay grades of 15%, the cost of the higher rates becomes so prohibitive that the minimum cost solution dictates that all input be in the pay grades E-3, E-4 and a smaller amount of E-5, (Figure C-8). At this point, since a certain number of personnel are needed to meet requirements, the same numbers of "cheap" personnel are brought into the program. Thus a further increase of relative cost would have no effect on the slope of the training input line.



500
490
480
470
460
450
440
430
570
550
530
510
490
470
450
430
FIVE-YEAR TOTAL TRAINING INPUT
FIVE-YEAR TOTAL COST
RELATIVE INCREASE IN COST
PER PAY GRADE
VS
TOTAL TRAINING INPUT
AND
TOTAL COST
LEGEND: TOTAL INPUT
TOTAL COST
0.00 0.05 0.10 0.15 0.20 0.25 0.30
ΔC
RELATIVE INCREASE IN COST PER PAY GRADE

FIGURE C-7



TRAINING INPUT VS YEAR OF INPUT
TRAINING INPUT VS PAY GRADE
A
B
TRAINING INPUT
300
200
100
0
YEAR OF INPUT
PAY GRADE
2
3
4
5
E3
E4
E5
E6
E7
ΔC=0.00
ΔC=0.05
ΔC=0.10
ΔC=0.15-0.30
ΔC=0.0
ΔC=0.05
ΔC=0.10
ΔC=0.15
ΔC=0.20
ΔC=0.25
ΔC=0.30
PROPORTIONAL INCREASE IN COST PER PAY GRADE

FIGURE C-8

5. Variation of Control Factor

The Control Factor of PIPE FIVE is a restraint on the number of personnel that the model may carry in excess of requirements during any one year in order to derive the minimum cost solution over a five year period of projection, (see Appendix D). Thus a control factor of 0. 0 requires the model to exactly meet requirements each year, whereas a control factor of 10% allows as much as 10% excess over requirements in any pay grade in any year as long as requirements are met. Control factors from 0. 0 to 50. 0% were used to test the sensitivity of the model to this parameter.

a. Figure C-9 demonstrates that the total training input does not vary as the control factor is increased and more excesses are allowed. This is because minimum requirements do not change, and must be met without regard to the control factor.

b. Figure C-9 illustrates that the total five-year cost decreases as the ceiling on excesses is relaxed. The reasons for this may be seen in Figure C-10. As more excesses are allowed in the training inputs, more E-3's are procured because they are cheaper than the E-4's, whose number decreases. The input of E-5's remains constant.

c. Figure C-11 shows the effect that an increase in control factor has on the year of input of the personnel. As more excesses are allowed, the number of people who enter in the fourth year of projection increases, while the number of those who enter in during the fifth year decreases.

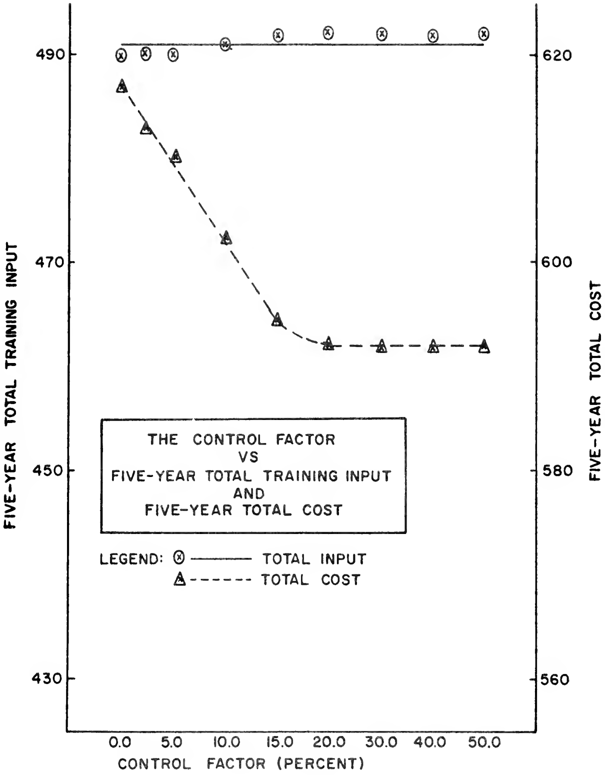

490
470
450
430
620
600
580
560
FIVE-YEAR TOTAL TRAINING INPUT
FIVE-YEAR TOTAL COST
THE CONTROL FACTOR
VS
FIVE-YEAR TOTAL TRAINING INPUT
AND
FIVE-YEAR TOTAL COST
LEGEND: ⊗ ——— TOTAL INPUT
Δ - - - - - - TOTAL COST
0.0 5.0 10.0 15.0 20.0 30.0 40.0 50.0
CONTROL FACTOR (PERCENT)

FIGURE C-9

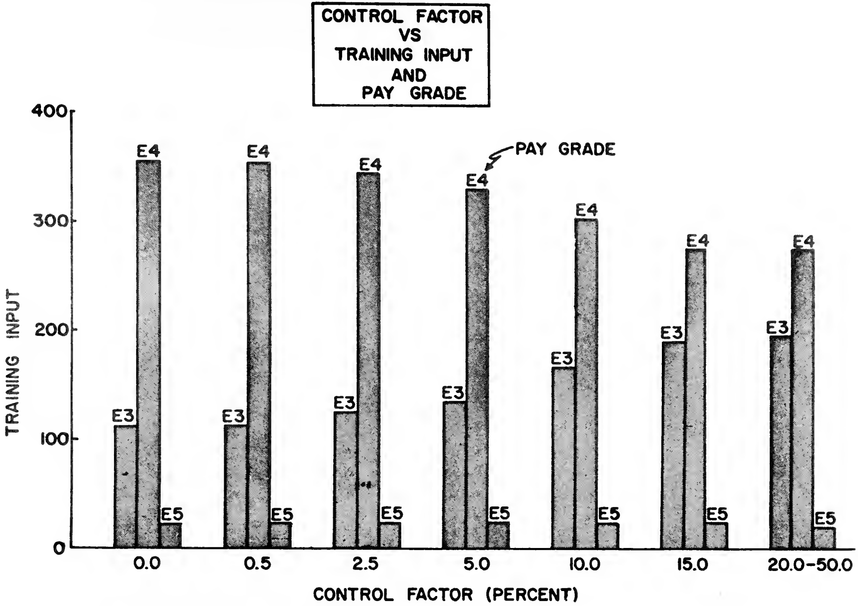

CONTROL FACTOR
VS
TRAINING INPUT
AND
PAY GRADE
TRAINING INPUT
400
300
200
100
0
PAY GRADE
E3
E4
E5
0.0
0.5
2.5
5.0
10.0
15.0
20.0-50.0
CONTROL FACTOR (PERCENT)

FIGURE C-10

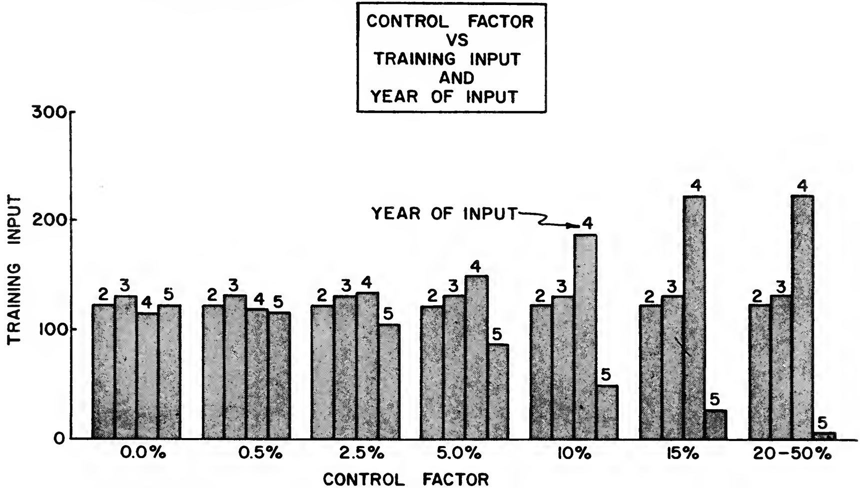

CONTROL FACTOR VS TRAINING INPUT AND YEAR OF INPUT
TRAINING INPUT
300
200
100
0
YEAR OF INPUT
2 3 4 5
0.0%
0.5%
2.5%
5.0%
10%
15%
20-50%
CONTROL FACTOR

FIGURE C-11

APPENDIX D

USER MANUAL

1. INTRODUCTION AND GENERAL DESCRIPTION

This Appendix is designed to provide the personnel planner with the information necessary to use the computer programs without a detailed knowledge of either the mathematical model from which they were derived or the techniques used in their construction.

The computer models described in this Appendix are the One Year Model (PIPE ONE), and the Five Year Model (PIPE FIVE). The One Year Model is essentially a mechanization of the techniques now used by personnel planners in BUPERS to project inventories using adding machines and desk calculators. The Five Year Model performs this same function but with two very important sophistications; a computational technique is used which:

a. allows the model to "look ahead" at future requirements and the necessary training inputs that these requirements demand;

b. adds these training inputs to the inventory over the five year period in such a manner that the total cost of these inputs is minimized while meeting requirements each year.

The ability of the computer to account for the future and to attach the notion of "cost" to the computed results are the significant additions to the planners' "bag of tools" offered by the Five Year Model.

The solution generated by a computer to a problem as complex as the planning of future enlisted inventories should not be regarded as a set of final precise figures, but rather as a guide to decision. The computer models are an attempt to simulate actual conditions (i. e., the natural environment) to a degree that will yield predictions accurate enough to make the application of the model worthwhile.

An objective in formulating these models was that they would produce a solution of accuracy comparable to the accuracy of the input data, and provide a rapid analytical means of solving enlisted projection problems using data familiar to the personnel planner.

2. INPUTS TO THE MODEL

This section discusses the inputs to the computer models which must be determined by the user. After the data has been assembled in the manner described in the following paragraphs it must be transferred to data sheets. Sample data sheets filled in with representative data for each computer input are provided. These data sheets are designed to assist the planner in preparing the data in a manner in which it can be readily interpreted by the computer key punch operators.

The inputs to the computer program are:

a. Initial Inventory

b. Requirements

c. Training Input Distribution

d. Cost Coefficients (only PIPE FIVE)

e. Attrition Rate

f. Retirement Rate

g. Reenlistment Rate

h. Promotion Rate

i. Control Factor (only PIPE FIVE)

j. Serial Number

These inputs are examined in detail in the following sections.

a. Initial Inventory: These models will project enlisted inventories of personnel with the following characteristics:

(1) similar rates of attrition, reenlistment, retirement and promotion;

(2) similar requirements to be met

For example, machinist mates and electronic technicians could conceivably satisfy the first criterion because of similar characteristics of promotion, attrition, reenlistment and retirement; however, they can not be projected as one group because they obviously do not fill similar billets.

Years Obligated Service	Pay Grade E-3	Pay Grade E-4		Pay Grade E-5		Pay Grade E-6			Pay Grade E-7			
	First Enlist-ment	First Enlist-ment	Second Enlist-ment	First Enlist-ment	Second Enlist-ment	First Enlist-ment	Second Enlist-ment	Third Enlist-ment	Second Enlist-ment	Third Enlist-ment	Fourth Enlist-ment	Fifth Enlist-ment
1	6	12	18	24	30	36	42	48	54	60	66	72
2	5	11	17	23	29	35	41	47	53	55	65	71
3	4	10	16	22	28	34	40	46	52	58	64	70
4	3	9	15	21	27	33	39	45	51	57	63	69
5	2	8	14	20	26	32	38	44	50	56	62	68
6	1	7	13	19	25	31	37	43	49	55	61	67

Figure II-3

Inventory Matrix

In order to be used by the computer the initial inventory must be subdivided by pay grade, obligated service, and current enlistment into a matrix of 72 cells as shown in Figure II-3, repeated here for the user's convenience. The six rows of the matrix represent years of obligated service and the twelve columns are divided among five pay grades. For the purpose of this matrix, E-1, E-2 and E-3 are compressed into one group, E-3. Similarly, E-7, E-8, and E-9 are considered together as E-7. Each column represents an enlistment, first, second, third, fourth or fifth. Note that there is provision for only one enlistment for E-3, two enlistments for E-4 and E-5, and three enlistments for E-6. The assumption was made that for critical ratings the number of personnel who did not fall within the limits of these pay grades would be inconsequential and could be neglected. Note also that there is no provision for E-7 in their first enlistment. The assumption is made that the numbers of people in this category (first enlistment chiefs) could be safely ignored.

The criterion for allocating individuals of the inventory into the appropriate rows and columns (i. e., cells) of the inventory matrix are given below:

a. Row Criterion

The row position of the members of the inventory is determined by the number of years of obligated service of an individual. Obligated service for this purpose is defined as the

difference between the calendar year of an individual's expiration of active obligated service and the present calendar year, plus one.

Let: n = (calendar year of expiration active obligated service)

- (present calendar year)

Then: row position = n + 1

Example: a person in a calendar year 1964 inventory whose expiration of active service occurs in calendar year 1967 would be placed in row number four. That is,

n = 1967 - 1964 = 3, so that

row position = n + 1 = 3 + 1 = 4

(2) Column Criterion

The column position of the members of the inventory is determined by their pay grade and current enlistment.

The following table summarizes the column breakdown:

COLUMN	PAY GRADE	CURRENT ENLISTMENT	LENGTH OF SERVICE IN YEARS
1	E-3*	First	0 to 6
2	E-4	First	0 to 6
3	E-4	Second	7 to 12
4	E-5	First	0 to 6
5	E-5	Second	7 to 12
6	E-6	First	0 to 6
7	E-6	Second	7 to 12
8	E-6	Third	13 to 18
9	E-7**	Second	7 to 12
10	E-7**	Third	13 to 18
11	E-7**	Fourth	19 to 24
12	E-7**	Fifth	25 to 30

Notes: * includes pay grades E-1, 2, and 3
** includes pay grades E-7, 8, and 9

After the inventory has been assembled in the manner described above, it must be transferred to data sheets. Figure D-1, Initial Inventory, is the data sheet used. The numbers above each box correspond to the column numbers on a standard 80-column IBM punch card onto which the item of data is punched. These numbers do not concern the planner. The appropriate data is placed in the boxes, one digit per box. There is one precaution in filling out the data sheets; whole numbers without decimal points must be right adjusted; percentages and decimal fractions are left adjusted. That is to say, a particular group of data boxes must be filled so that there are no blanks on the right or left as the case may be. A blank box will be interpreted as a zero by the computer. In both right and left adjusted data the user may indicate the proper location of the decimal by placing a decimal point in the appropriate box, keeping in mind that blanks are read as zeros.

A completed Initial Inventory Data Sheet, Figure D-1, represents one column of the inventory matrix. Twelve of these data sheets are required for the initial inventory, each sheet representing a column of the inventory matrix. The data are placed in the boxes as shown in the figure.

Column No. 4 INITIAL INVENTORY - INPUT DATA SHEET Rate ______

Pay Grade E-4

	8	9	10	11	12		20	21	22	23	24
6 YEARS OBLIGATED SERVICE				2	6	5 YEARS OBLIGATED SERVICE				3	2

	32	33	34	35	36		44	45	46	47	48
4 YEARS OBLIGATED SERVICE				2	1	3 YEARS OBLIGATED SERVICE				1	6

	56	57	58	59	60		68	69	70	71	72
2 YEARS OBLIGATED SERVICE				1	8	1 YEAR OBLIGATED SERVICE				1	3

NOTE: 1) Twelve data sheets are required for a complete Initial Inventory.

2) All data on this sheet must be right adjusted.

FIGURE D-1

b. Requirements: As used by the computer programs requirements must:

(1) be expressed by pay grade and,

(2) correspond to the initial inventory.

These requirements, by pay grade, must be defined for each year of projection, and may differ from year to year.

A completed Requirements Data Sheet for a five year projection is found in Figure D-2. One completed data sheet is required for a five year projection.

c. Training Input Distribution: As stated in the introduction to this Appendix, one of the outputs of the computer program is the "training inputs" which are required to maintain the projected inventory at a strength necessary to meet requirements. The distribution of the training input in the inventory matrix, i. e., to which cells this training input will actually be added is determined by the personnel planner. He must identify the amount of obligated service and the enlistment number for the computed training input to each pay grade. The model is constructed so that input may be made to any cell which represents personnel with less than 18 years of service, (all columns except 11 and 12). The user must determine for each pay grade and for each column within each pay grade:

(1) If there will be a training input in a particular column and,

REQUIREMENTS - INPUT DATA SHEET

Rate ________

YEAR OF PROJECTION	E-3 (2–6)	E-4 (8–12)	E-5 (14–18)	E-6 (20–24)	(26–30)
1	26	128	177	102	76
2	31	149	210	120	89
2	33	166	231	133	99
3	36	178	248	142	106
5	36	178	248	142	106

NOTE: 1) All data must be right adjusted.

FIGURE D-2

(2) If there is an input into which particular cells of the column the input will be allowed.

Once the training input distribution has been established the computer solves the problem for the number of people who must be added to a particular pay grade to maintain requirements, and then divides this input equally among the cells of the pay grade that have been designated by the user.

Looking at the data input sheet, Figure D-3, for the Training Input Distribution, the terms "upper" and "lower" limits of a cell must be defined. The "upper" limit is the largest cell number in the column into which training input will be added, and the "lower" limit is the smallest cell number. For example, if in column #3 the training input is to be placed in cells 13, 14, 15 and 16, then the upper limit would be 16 and the lower limit 13. In determining the limits for any particular column there is the restriction that in any one column the cells into which training input may be assigned must be consecutive. It is important that each limit be filled either with a number corresponding to a cell number or a zero, which indicates no input. If input is to be allowed in only one cell of a particular column then this cell number must appear as the "lower" limit and a zero must be entered as the "upper" limit.

TRAINING INPUT DISTRIBUTION - INPUT DATA SHEET

Rate ________

PAY GRADE		LOWER LIMIT	UPPER LIMIT		LOWER LIMIT	UPPER LIMIT		LOWER LIMIT	UPPER LIMIT
		2 3	5 6						
E-3	COLUMN ONE	2	5						
		2 3	5 6		8 9	11 12			
E-4	COLUMN TWO	9	11	COLUMN THREE	14	16			
		2 3	5 6		8 9	11 12			
E-5	COLUMN FOUR	20	23	COLUMN FIVE	26	29			
		2 3	5 6		8 9	11 12		14 15	17 18
E-6	COLUMN SIX	0	0	COLUMN SEVEN	38	41	COLUMN EIGHT	44	47
		2 3	5 6		8 9	11 12			
E-7	COLUMN NINE	50	53	COLUMN TEN	56	59			

NOTE: 1) A separate IBM data card is required for each pay grade.

2) All data must be right adjusted.

3) If only one input cell is to be allowed in any column the appropriate cell number must appear in the lower limit.

FIGURE D-3

The following examples indicate how these cell limits are determined and also suggest various training input distributions that might be used:

Example (1): Training input is to be allowed in all grades, but the input must have four or more years of obligated service. The limits for the respective columns are:

Column Number	Lower Limit	Upper Limit
1	1	3
2	7	9
3	13	15
4	19	21
5	25	27
6	31	33
7	37	39
8	43	45
9	49	51
10	55	57

Example (2): Training input is to be allowed in the first enlistment only and the input must have more than four years of obligated service. The limits for the respective columns are:

Column Number	Lower Limit	Upper Limit
1	1	2
2	7	8
3	0	0
4	19	20
5	0	0
6	31	32
7	0	0
8	0	0
9	0	0
10	0	0

Example (3): Training input is to be allowed in the first enlistment and the input must have more than four years of obligated service except that an E-6 with 6 years of obligated service may be accepted. The limits of the respective columns are the same as in Example (2) above except for column #7 where the lower limit would be 37 and the upper limit would be 0.

Example (4): Training input is to be allowed for personnel with between 4 and 8 years of service. The limits for the respective columns are:

Column Number	Lower Limit	Upper Limit
1	4	6
2	10	12
3	13	14
4	22	24
5	25	26
6	34	36
7	37	38
8	0	0
9	49	50
10	0	0

Example (5): No training input will be allowed for E-6 and E-7. (This might be a situation where no matter what solution the computer might generate which says "add E-6 and E-7" these personnel may be unavailable and hence such a solution is unacceptable.) If the training input to a particular grade is suppressed the user must accept the possibility that the numerical requirements for this particular grade may not be met; however, overall numerical requirement for

the inventory will always be met, provided that the input to the lowest grade E-3 is not that pay grade that is suppressed. In this example, if the requirements for E-6 and E-7 are not met they will be numerically filled by E-5 and lower pay grades. For this example, the "Lower" and "Upper" limits for columns #6 through #10 would be zero.

d. Cost Coefficients: The cost coefficients provide a means whereby the computer model can consider the relative costs of adding personnel of different pay grades to the inventory. The cost coefficients measure the relative difficulty of obtaining personnel of one pay grade as opposed to another. They may be expressed in terms of dollars, or as ratios which represent procurement difficulties. If expressed in terms of dollars, the planner may consider training costs, pay and allowances, etc. In the process of solving for the training input to meet requirements, the computer considers the cost of adding this input to the various pay grades and then solves for the least expensive solution, based on the cost coefficients determined by the user.

Five cost coefficients are required, one for each pay grade. Figure D-4 is a sample cost coefficient data sheet completed with the cost coefficients used by the authors in their evaluation of the Five Year Model. Note that these coefficients may use the decimal

COST COEFFICIENTS, CONTROL FACTOR, AND SERIAL NUMBER

Rate ___________

COST COEFFICIENTS

PAY GRADE E-3

1	2	3	4	5	6
	1	.	0	0	0

PAY GRADE E-4

7	8	9	10	11	12
	1	.	3	2	0

PAY GRADE E-5

13	14	15	16	17	18
	1	.	5	8	0

PAY GRADE E-6

19	20	21	22	23	24
	1	.	8	7	0

PAY GRADE E-7

25	26	27	28	29	30
	2	.	3	2	0

CONTROL FACTOR

4	5	6	7
	1	0	.

SERIAL NUMBER

4	5	6	7
			1

NOTE: 1) Cost coefficients and Control Factor must be either left adjusted or a decimal point indicated.
2) Serial Number must be right adjusted, no decimal point allowed.

FIGURE D-4

point. If the data is expressed in decimal form, as in the example, Figure D-4, the user must place the decimal point in the proper box. If no decimal is used, the numbers must be right adjusted as explained previously.

e. Attrition Rate: Associated with the passage of each increment of time is the likelihood that a certain fraction of the inventory will be lost or "attrited" due to unforseen circumstances. In order to account for this "unplanned" loss of personnel, an attrition rate is applied to the inventory in the model. This rate is composed of the summation of specific types of losses expressed as a percentage reduction to be applied to the inventory each year.

Attrition rate consists of the following types of personnel losses: (The numbers in parentheses following the various types of losses are the specific item numbers for attrition of active duty enlisted personnel as defined in NAVPERS 15658, <u>Navy and Marine Corps Military Personnel Statistics</u>.)

(1) Death (#1290)
(2) Physical Disability (#1120)
(3) Discharge by Undesirable Discharge (#1000)
(4) Discharge by Bad Conduct Discharge (#1040)
(5) Discharge by Dishonorable Discharge (#1050)
(6) Discharge by Reason of Dependency or Hardship (#890)
(7) Intra-Navy Transfer (#1250)
(8) Miscellaneous

The attrition rate data sheet, Figure D-5, shows that an attrition rate is entered for each cell of the matrix. The attrition

Column No. 7

ATTRITION RATE - INPUT DATA SHEET

Rate ______

Pay Grade E-6

	8	9	10	11	12
6 YEARS OBLIGATED SERVICE	1	5	0		

	20	21	22	23	24
5 YEARS OBLIGATED SERVICE	1	5	0		

	32	33	34	35	36
4 YEARS OBLIGATED SERVICE	1	5	0		

	44	45	46	47	48
3 YEARS OBLIGATED SERVICE	1	5	0		

	56	57	58	59	60
2 YEARS OBLIGATED SERVICE	1	5	0		

	68	69	70	71	72
1 YEAR OBLIGATED SERVICE	1	5	0		

NOTE: 1) Twelve data sheets are required of a complete set of attrition rate input data.

2) All data must be either left adjusted or a decimal point indicated.

FIGURE D-5

rate is a decimal fraction which is always less than one. In entering the data the user may either indicate the position of the decimal point or the data must be left adjusted so that it is properly interpreted by the computer.

f. Retirement Rate: The retirement rate is defined as the percentage of personnel in the inventory who are lost through voluntary retirement by reason of having completed 20 or more years of active service. Since only 11 cells of the inventory, (in pay grade E-7, cells 62 through 72), represent a total length of service of 20 years or more, only those cells have an associated retirement rate. A retirement rate must be assigned to each of these 11 cells. For example, if fifty percent of the E-7's who start the twentieth year are expected to retire within one year then the retirement rate for 62 would be 0.5 (point five). A completed sample retirement data sheet is shown in Figure D-6.

g. Reenlistment Rate: Reenlistment rate is the ratio of the number of men reenlisting to those eligible to reenlist, expressed as a percentage. The reenlistment rate is applied at the termination of a person's obligated service. Looking at the inventory matrix in Figure II-3, the top row represents personnel with one year of obligated service. In each year of projection the personnel in this row are subject to a reenlistment rate. The percentage who reenlist are added to the inventory of the next higher enlistment in the cell

RETIREMENT RATE - INPUT DATA SHEET

Rate ________

Pay Grade E-7

	8	9	10	11	12		20	21	22	23	24		32	33	34	35	36
CELL NUMBER 61		.	0	0	0	CELL NUMBER 62		.	5	0	0	CELL NUMBER 63		.	2	5	0

	44	45	46	47	48		56	57	58	59	60		68	69	70	71	72
CELL NUMBER 64		.	5	0	0	CELL NUMBER 65		.	5	0	0	CELL NUMBER 66		.	5	0	0

	8	9	10	11	12		20	21	22	23	24		32	33	34	35	36
CELL NUMBER 67		.	5	0	0	CELL NUMBER 68		.	5	0	0	CELL NUMBER 69		.	7	5	0

	44	45	46	47	48		56	57	58	59	60		68	69	70	71	72
CELL NUMBER 70		.	7	5	0	CELL NUMBER 71		.	7	5	0	CELL NUMBER 72		.	7	5	0

NOTE: 1) All data must either be left adjusted or a decimal point indicated.

FIGURE D-6

which represents 6 years of obligated service; the percentage that fail to reenlist are lost from the inventory. For personnel in any cell except those in the top row of the inventory matrix the value of the reenlistment rate is 100%.

The planner must determine twelve reenlistment rates; E-3 at the end of the first enlistment, E-4 and E-5 at the end of the first and second enlistments, E-6 at the end of the first, second, and third enlistments, and E-7 at the end of the second, third, fourth, and fifth enlistments. One completed data sheet is required for each projection as per the sample, Figure D-7.

1. Promotion Rate: Promotion rate is defined as the percentage of men promoted from one enlisted pay grade to the next higher enlisted pay grade in one year. Promotion is limited to intra-enlisted promotions, and since the inventory matrix considers E-7, E-8 and E-9 as one group, (E-7), only cells in pay grades E-6 and junior have an associated promotion rate. Furthermore, pay grades E-1, E-2 and E-3 are compressed into a single pay grade, E-3, so that the lowest promotions are to E-4. Promotion rates for the individual cells may be determined from statistical data of past promotions, from current promotion plans or from any other similar source.

Except for cells 31 through 35, all cells below cell number 62 are assigned a promotion rate between and including zero to 100%.

REENLISTMENT RATE - INPUT DATA SHEET

Rate ____________

E-3 END 1st ENL.	8	9	10	11	12
		.	2	5	4

E-4 END 1st ENL.	20	21	22	23	24
		.	2	5	4

E-4 END 2nd ENL.	32	33	34	35	36
		.	8	5	6

E-5 END 1st ENL.	44	45	46	47	48
		.	2	5	4

E-5 END 2nd ENL.	56	57	58	59	60
		.	8	5	6

E-6 END 1st ENL.	68	69	70	71	72
		.	2	5	4

E-6 END 2nd ENL.	8	9	10	11	12
		.	9	0	2

E-6 END 3rd ENL.	20	21	22	23	24
		.	9	5	0

E-7 END 2nd ENL.	32	33	34	35	36
		.	9	5	0

E-7 END 3rd ENL.	44	45	46	47	48
	1	.	0	0	0

E-7 END 4th ENL.	56	57	58	59	60
	1	.	0	0	0

E-7 END 5th ENL.	68	69	70	71	72
		.	0	0	0

NOTE: 1) All data must either be left adjusted or a decimal point indicated.

FIGURE D-7

In general, promotion is between cells in the same enlistment, i. e., an E-4 in the first enlistment is promoted to a cell in pay grade E-5 which is in the first enlistment column of that pay grade. (The only exception is at the reenlistment point where a may may be considered to be promoted and reenlisted at the same time in which case he would move to the next higher pay grade in the next higher enlistment column.) Cells 31 to 35 which represent E-6 in their first enlistment have zero rates of promotion because pay grade E-7 in the model starts in the second enlistment; hence movement from the first enlistment of E-6 is impossible.

Figure D-8 is a completed sample Promotion Rate data sheet. Eight data sheets are required, one for each column.

i. Control Factor: As explained in paragraph d., "Cost Coefficients," the computer solution for the training input to meet requirements is a minimum cost solution. This means that the computer has solved the problem without regard to how many people are added in any one year except for the restriction that the numerical requirements must be satisfied. Therefore, unless the computer is somehow restrained it will not consider the actual yearly distribution of the input, nor will there be any limit on the excess of requirements that may occur in any one year of the projection.

Column No. 2 PROMOTION RATE - INPUT DATA SHEET Rate ______

Pay Grade E-4

	8	9	10	11	12
6 YEARS OBLIGATED SERVICE	0	0	0		

	20	21	22	23	24
5 YEARS OBLIGATED SERVICE	4	0	0		

	32	33	34	35	36
4 YEARS OBLIGATED SERVICE	6	0	0		

	44	45	46	47	48
3 YEARS OBLIGATED SERVICE	6	0	0		

	56	57	58	59	60
2 YEARS OBLIGATED SERVICE	6	0	0		

	68	69	70	71	72
1 YEAR OBLIGATED SERVICE	6	0	0		

NOTE: 1) Twelve data sheets are required for a complete set of Promotion Rate input data sheets.

2) All data must be either left adjusted or a decimal point indicated.

FIGURE D-8

The minimum cost solution for the overall period is simply the least expensive means of meeting requirements, through training input, without regard to the resultant distribution of the inventory in any particular year. As an example, the minimum cost solution for the five year period may call for all the total input to be affected in the first two years. Conceptually this answer may be correct, as this may be the least expensive solution. However, as a practical procedure this solution may be unworkable because an even flow over the five year span may be required by the training facility capacity. Furthermore, the requirements in this example would probably be greatly exceeded in the first two years, tapering off to just meeting requirements in the fifth year.

To limit the number of personnel in any one year of the projection that may be in excess of requirements, an input called the "control factor" is provided. This is a number which controls the amount by which requirements may be exceeded in solving for a minimum cost solution. The control factor is defined as the percentage of the total requirements by which the inventory may exceed requirements in any one year. For example, if the control factor is 10% and total requirements are for 800 personnel (i. e., in all pay grades), the solution is limited to a total of 880 people in the inventory even if a more economical solution would dictate a greater number in a particular year.

The smaller the control factor, the more restrictive and expensive will be the solution. The most restrictive value of the control factor is zero, which causes the computer to fill requirements as vacancies occur. This is the maximum cost solution in that the ability of the model to anticipate future losses is not utilized and the advantage of flexibility of solution has been removed. It was determined through analysis that using a control factor of 100% effectively removes any restriction on the amount that inputs may exceed requirements. In using the five year model it is recommended that a control factor of 100% be used to first find the minimum cost solution. If this solution is not satisfactory with respect to yearly input the control factor can be adjusted so that the requirements in any one year will not be excessive. The control factor input is illustrated in Figure D-4.

j. Serial Number: This input provides the user with a means of serializing a computer run. This number may take any value from 1 to 9999 but may not be a decimal. This input is entered on the data sheet for "Cost Coefficients", Figure D-4

3. OUTPUTS OF THE MODEL

The basic outputs of the model are:

a. Projected inventory;

b. Training input;

c. Training input cost, (PIPE FIVE only).

The computer is programmed to display these outputs in two forms, Yearly Inventory Summary, Figure D-9 and Five Year Summary, Figure D-10. These two output sheets are summarized in the following paragraphs.

a. Yearly Summary, Figure D-9.

(1) In the left upper corner "RUN NO. 1", the numerical value is the run "index" or serial number which is a result of input number 10.

(2) "INVENTORY PROJECTION YEAR NO. __" is the year of projection that is represented by the inventory matrix. "Year No. 1" would represent the inventory at the end of the first projection year.

(3) "TOTAL PROJECTED INVENTORY" represents the pay grade totals and grand total of the inventory matrix.

(4) "TOTAL TRAINING INPUTS" summarizes the training inputs by pay grade and indicates an overall total.

(5) "EXCESS (SHORTAGE) OF REQ" indicates how the total projected inventory for a pay grade compares with requirements for that grade. A positive number indicates an excess of requirements, where

RUN NO. 1

INVENTORY PROJECTION YEAR NO. 3

PAY GRADE

OBSERV	E 3	E 4	E 4	E 5	E 5	E 6	E 6	E 6	E 7	E 7	E 7	E 7
1	1.8	12.8	.1	22.9	2.6	23.0	11.7	4.4	11.7	21.4	3.1	.0
2	8.5	33.2	5.9	59.3	13.2	27.6	9.1	1.0	13.1	12.4	.7	.0
3	8.2	36.9	20.5	64.6	11.6	8.4	3.6	1.4	8.7	16.8	.7	.0
4	8.2	19.9	20.6	2.5	12.3	.0	2.5	.9	1.1	1.1	.C	.5
5	6.4	.0	15.1	2.2	5.0	.0	4.4	6.8	1.1	9.7	5.2	.8
6	.0	.0	1.0	.0	5.2	.0	8.6	10.5	.C	13.6	14.1	1.9

TOTAL PROJECTED INVENTORY

PG E-3	33.0
PG E-4	166.0
PG E-5	201.5
PG E-6	123.8
PG E-7	137.6
TOTAL	662.0

TOTAL TRAINING INPUTS

PG E-3	25.5
PG E-4	87.3
PG E-5	17.8
PG E-6	.0
PG E-7	.0
TOTAL	130.6

EXCESS (SHORTAGE) OF REQ

PG E-3	-.0
PG E-4	-.0
PG E-5	-29.5
PG E-6	-9.2
PG E-7	38.6
TOTAL	.0

TRAINING INPUT PER CELL

PAY GRADE

OBSERV	E 3	E 4	E 4	E 5	E 5	E 6	E 6	E 6	E 7	E 7	E 7	E 7
1	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
2	6.4	14.6	.0	2.2	2.2	.0	.0	.0	.0	.0	.C	.0
3	6.4	14.6	14.6	2.2	2.2	.0	.0	.0	.0	.0	.0	.0
4	6.4	14.6	14.6	2.2	2.2	.0	.0	.0	.0	.0	.C	.0
5	6.4	.0	14.6	2.2	2.2	.0	.0	.0	.0	.0	.0	.0
6	.0	.0	.0	.0	.C	.0	.0	.0	.0	.0	.0	.0

FIGURE D -9

YEARLY SUMMARY

FIVE YEAR SUMMARY STUDY NUMBER 1

YEAR	1	2	3	4	5
		EXCESS (SHORTAGE) OF REQ			
PG E-3	-16.9	-.0	-.0	112.5	.1
PG E-4	-11.5	-.0	-.0	-.5	-.2
PG E-5	-5.1	-40.8	-29.5	-45.8	-67.5
PG E-6	11.5	3.1	-9.2	11.0	16.7
PG E-7	44.0	37.7	38.6	40.6	48.3
TOTAL	22.0	-.0	.0	117.8	-2.6
		TOTAL PROJECTED INVENTORY			
PG E-3	9.1	31.0	33.0	148.5	36.1
PG E-4	116.5	149.0	166.0	177.5	177.8
PG E-5	171.9	169.2	201.5	202.2	180.5
PG E-6	113.5	123.1	123.8	153.0	158.7
PG E-7	120.0	126.7	137.6	146.6	154.3
TOTAL	531.0	599.0	662.0	827.8	707.4
		TOTAL TRAINING INPUTS			
PG E-3	.0	29.1	25.5	140.8	.0
PG E-4	.0	90.5	87.3	92.4	5.8
PG E-5	.0	2.5	17.8	.0	.0
PG E-6	.0	.0	.0	.0	.0
PG E-7	.0	.0	.0	.0	.0
TOTAL	.0	122.1	130.6	233.2	5.8

TOTAL FIVE YEAR TRAINING INPUT 491.7

TOTAL FIVE YEAR COST 591.7

COST COEFICIENTS 1.000 1.320 1.580 1.870 2.320

FIGURE D-10

FIVE YEAR SUMMARY

the projected total does not meet requirements, a minus number indicates the shortage. Only under most unusual circumstances will the actual inventory precisely fit the requirements structure. It is recalled that the computer is programmed to meet total requirements from the "top" down so that a shortage in E-3, for example, is made up for by an excess in a higher pay grade.

(6) "TRAINING INPUT PER CELL" shows the training input into the inventory matrix by cell.

b. Five Year Summary, Figure D-10. This Output Form repeats the EXCESS (SHORTAGE) OF REQ, TOTAL PROJECTED INVENTORY, and TOTAL TRAINING INPUTS totals from the one year summaries. The TOTAL FIVE YEAR TRAINING INPUT is the summation of the yearly training totals. The FIVE YEAR COST is the total cost of the training input. It is computed by multiplying the total five year training input for a pay grade by the cost coefficient for that pay grade, and summing over all pay grades.

4. SPEED OF COMPUTATION (CDC 1604

a. PIPE ONE; 50 seconds to compile, then one second for each year of projection.

b. PIPE FIVE; 2 minutes 50 seconds to compile, then 15 seconds for each year of projection.

It is obvious that one of the major advantages of these computer programs is their speed. A planner faced with selecting a promotion rate for a particular grade can evaluate many plans and their affects in a very short period of time. The most efficient use of the computer, because of the relatively long time required by the compiling processes, is to make as many projections as possible at one time.

5. USES FOR PIPE ONE

a. PIPE ONE projects inventories for one year, computes the training input to meet total requirements (filling senior requirements first), and may be cycled for any number of years to give a long range projection. The advantages of this program are that it takes little computer time and can be used to make projections beyond five years. The disadvantages when compared to PIPE FIVE are that it does not give the "cost" information and it merely fills vacancies as they occur.

b. PIPE ONE, with a minor program change, can be made to project inventories with a constant training input into any cell or group of cells of the inventory matrix. In this manner proposed training input policies may be evaluated.

c. The effects of various promotion and retirement policies on the projected inventories and training inputs may be evaluated.

d. The effects of an increase (or decrease) in reenlistment rates may be evaluated.

e. PIPE ONE, with a minor program change, can be used to build an enlisted "model structure" for any number of years for any particular set of promotion, attrition, reenlistment and retirement rates. This may be accomplished by a program change which would provide a training input of 100 people in cell #1 in each year of projection. Initial inventory and requirements are set at zero. The projected inventory for the last year of projection will show for each cell the percentage of the initial inventory, i. e., 100 people still on board after the passage of time.

6. USES FOR PIPE FIVE

a. PIPE FIVE projects inventories for five years, computes the training input so as to meet total requirements in the least expensive manner and determines the cost of this training input. This program takes into account future losses and changes in requirements in computing the training input. The program anticiaptes future needs while meeting current requirements and hence may compute a training input that causes the total requirements to be exceeded in one or more of the years of projection. The amount that requirements may be exceeded in any year may be controlled by the user.

b. As in PIPE ONE, PIPE FIVE can evaluate the effects of promotion rate, reenlistment, and retirement rate on the inventory and on the training inputs required to insure that the inventory meets requirements. At the same time it can evaluate the changes in the cost of the training input for various changes in these rates.

c. By adjusting the "control factor" the planner can examine a range of "cost" solutions and use the solution which is the best compromise between training cost, uniform yearly training input, and the number of people in excess of requirements.

APPENDIX E

COMPUTER PROGRAM LISTINGS

1. PIPE ONE

The following listing for PIPE ONE is written in FORTRAN 60.

```
      PROGRAM PIPE CNE

     0DIMENSION X(72),ALFA(72),BETA(72),GAMMA(72),DELTA(72),
     1P(5,72),T(5,72),TAU(5,5),REC(5,5),R(5,5),TEMP(5,6),
     2C(5,72),D(5,72),CK(5,72),SP(5,6),ST(5,6),EXR(5,6),
     3TX(5),TA(10),L(20)
     0COMMON X,ALFA,BETA,GAMMA,CELTA,P,T,TAU,REQ,R,TEMP,
     1C,D,CK,SP,ST,EXR,M,NR,TX,TA,L

      READ INPUT TAPE 5,1100,(X(N),N=1,72)
      READ INPUT TAPE 5,1200,((REC(M,J),J=1,5),M=1,5)
      READ INPUT TAPE 5,1030,L(1),L(2)
      READ INPUT TAPE 5,1040,(L(I),I=3,6)
      READ INPUT TAPE 5,1040,(L(I),I=7,10)
      READ INPUT TAPE 5,1050,(L(I),I=11,16)
      READ INPUT TAPE 5,1040,(L(I),I=17,20)
      READ INPUT TAPE 5,1300,NR
      READ INPUT TAPE 5,1100,(ALFA(N),N=1,72)
      READ INPUT TAPE 5,1100,(BETA(N),N=61,72)
      READ INPUT TAPE 5,1100,(CELTA(N),N=6,72,6)
      READ INPUT TAPE 5,1100,(GAMMA(N),N=1,72)

 1030 FORMAT (I3,I3)
 1040 FORMAT (I3,I3,I3,I3)
 1050 FORMAT (I3,I3,I3,I3,I3,I3)
 1100 FORMAT (6F12.4)
 1200 FORMAT (5F6.0)
 1300 FORMAT (I4)
```

```
C   MAIN PROGRAM   PIPE ONE

      CALL HOUSE ONE

      DO 4800 M=1,5
      CALL VENT ONE
      CALL TRAIN
C COMPUTE PROJECTED INVENTORY
      DO 4600 N=1,72
 4600 P(M,N)=CK(M,N)+T(M,N)

      CALL TOTAL (M,P,SP)
      CALL TOTAL(M,T,ST)
      CALL EXCESS
      CALL OUTPUT

      DO 4700 N=1,72
 4700 X(N)=P(M,N)
 4800 CONTINUE

      STOP

      END
```

```
      SUBROUTINE TRAIN ONE

     ODIMENSION X(72),ALFA(72),BETA(72),GAMMA(72),DELTA(72),
     1P(5,72),T(5,72),TAU(5,5),REQ(5,5),R(5,5),TEMP(5,6),
     2C(5,72),D(5,72),CK(5,72),SP(5,6),ST(5,6),EXR(5,6),
     OCOMMON X,ALFA,BETA,GAMMA,DELTA,P,T,TAU,REQ,R,TEMP,
     1C,D,CK,SP,ST,EXR,M,NR,TX,TA,L
C COMPUTE  TRAINING INPUT PER PAY GRADE  TAU(M,N)

      NI=1
      DO 3200 J=1,5
      SUM=0.0
      DO 2400 N=NI,72
 2400 SUM=SUM+CK(M,N)
      IF(R(M,J)-SUM)2500,2500,2600
 2500 TEMP(M,J)=0.0
      GO TO 2700
 2600 TEMP(M,J)=R(M,J)-SUM
 2700 GO TO (2800,2900,3000,3100,3200),J
 2800 NI=7
      GO TC 3200
 2900 NI=19
      GO TO 3200
 3000 NI=31
      GO TO 3200
 3100 NI=49
 3200 CONTINUE
```

```
      TEMP(M,6)=0.0
      DO 3500 J=1,5
      IF(TEMP(M,J)-TEMP(M,J+1))3300,3300,3400
 3300 TAU(M,J)=0.0
      GO TO 3500
 3400 TAU(M,J)=TEMP(M,J)-TEMP(M,J+1)
 3500 CONTINUE
      K=1
      JA=1
      JB=2
      DO 45 I=1,10
      IF(L(JA))10,10,15
   15 IF(L(JB))20,20,25
   20 T(M,JA)=TAU(M,K)*TX(K)
      GO TO 10
   25 MA=L(JA)
      MB=L(JB)
      DO 30 N=MA,MB
   30 T(M,N)=TAU(M,K)*TX(K)
   10 GO TO (40,44,41,44,42,44,44,43,44,44),I
   40 K=2
      GO TO 44
   41 K=3
      GO TO 44
   42 K=4
      GO TO 44
   43 K=5
   44 JA=JA+2
   45 JB=JB+2
      RETURN
      END
```

```
      SUBROUTINE FACTOR
     0DIMENSION X(72),ALFA(72),BETA(72),GAMMA(72),DELTA(72),
     1P(5,72),T(5,72),TAU(5,5),REQ(5,5),R(5,5),TEMP(5,6),
     2C(5,72),D(5,72),CK(5,72),SP(5,6),ST(5,6),EXR(5,6),
     3TX(5),TA(10),L(20)
     0COMMON X,ALFA,BETA,GAMMA,DELTA,P,T,TAU,REQ,R,TEMP,
     1C,D,CK,SP,ST,EXR,M,NR,TX,TA,L

      JA=1
      JB=2
      DO 30 I=1,10
      TA(I) = 0.0
      IF(L(JA))10,10,15
   15 IF(L(JB))20,20,25
   20 TA(I)=1
      GO TO 10
   25 TA(I)=L(JB)-L(JA)+1
   10 JA=JA+2
   30 JB=JB+2
      IF(TA(1))61,61,60
   60 TX(1)=1./TA(1)
   61 IF(TA(2)+TA(3))63,63,62
   62 TX(2)=1./(TA(2)+TA(3))
   63 IF(TA(4)+TA(5))65,65,64
   64 TX(3)=1./(TA(4)+TA(5))
   65 IF(TA(6)+TA(7)+TA(8))67,67,66
   66 TX(4)=1./(TA(6)+TA(7)+TA(8))
   67 IF(TA(9)+TA(10))69,69,68
   68 TX(5)=1./(TA(9)+TA(10))
   69 CONTINUE
      RETURN
      END
```

```
      SUBRCUTINE EXCESS
     ODIMENSION X(72),ALFA(72),BETA(72),GAMMA(72),DELTA(72),
     1P(5,72),T(5,72),TAU(5,5),REQ(5,5),R(5,5),TEMP(5,6),
     2C(5,72),D(5,72),CK(5,72),SP(5,6),ST(5,6),EXR(5,6),
     3TX(5),TA(10),L(20)
     OCOMMON X,ALFA,BETA,GAMMA,DELTA,P,T,TAU,REQ,R,TEMP,

     1C,D,CK,SP,ST,EXR,M,NR,TX,TA,L
C COMPUTE EXCESS OF REQUIREMENTS BY PAY GRADE

      DO 8500 J = 1,5
      EXR(M,J) = SP(M,J) - REQ(M,J)
 8500 CONTINUE
      SUM=0.0
      DO 8600 J = 1,5
 8600 SUM = SUM + EXR(M,J)
      EXR(M,6) = SUM
 8700 CONTINUE
      RETURN
      END
```

```
      SUBRCUTINE VENT ONE

     0DIMENSION X(72),ALFA(72),BETA(72),GAMMA(72),DELTA(72),
     1P(5,72),T(5,72),TAU(5,5),REC(5,5),R(5,5),TEMP(5,6),
     2C(5,72),D(5,72),CK(5,72),SP(5,6),ST(5,6),EXR(5,6),
     3TX(5),TA(10),L(20)
     0COMMON X,ALFA,BETA,GAMMA,CELTA,P,T,TAU,REQ,R,TEMP,
     1C,D,CK,SP,ST,EXR,M,NR,TX,TA,L

C COMPUTE  CONSTANT C(M,N)
      C(M,1)=0.0
      DO 1700 N=2,72
 1700 C(M,N)=X(N-1)*(1.0-(ALFA(N-1)+BETA(N-1)))

C COMPUTE CONSTANT D(M,N)
      DO 1800 N=1,72
 1800 D(M,N)=0.0
      DO 1900 N=8,13
 1900 D(M,N)=DELTA(N-7)*GAMMA(N-7)*C(M,N-6)
      DO 2000 N=20,30
 2000 D(M,N)=DELTA(N-13)*GAMMA(N-13)*C(M,N-12)
      DO 2100 N=32,43
 2100 D(M,N)=DELTA(N-13)*GAMMA(N-13)*C(M,N-12)
      DO 2200 N=49,61
 2200 D(M,N)=DELTA(N-13)*GAMMA(N-13)*C(M,N-12)

C COMPUTE CONSTANT CK(M,N)
      DO 2300 N=1,72
 2300 CK(M,N)=D(M,N)+(C(M,N)*DELTA(N-1)*(1.0-GAMMA(N-1)))
      CK(M,1)=0.0
      CK(M,7)=0.0
      CK(M,19)=0.0
      CK(M,31)=0.0
      CK(M,49)=D(M,49)
      RETURN
      END
```

```
      SUBRCUTINE HOUSE ONE

     ODIMENSION X(72),ALFA(72),BETA(72),GAMMA(72),DELTA(72),
     1P(5,72),T(5,72),TAU(5,5),REQ(5,5),R(5,5),TEMP(5,6),
     2C(5,72),D(5,72),CK(5,72),SP(5,6),ST(5,6),EXR(5,6),
     3TX(5),TA(10),L(20)

     OCOMMON X,ALFA,BETA,GAMMA,CELTA,P,T,TAU,REQ,R,TEMP,
     1C,D,CK,SP,ST,EXR,M,NR,TX,TA,L

      DO 1400 N=1,72
 1400 ALFA(N) = 0.015
      DO 1500 N=1,60
 1500 BETA(N)=0.0

C COMPUTE CUMMULATIVE REQUIREMENTS R(M,J)
      JS=6
      SUM=0.0
      DO 1600 K=1,5
      SUM=SUM+REQ(M,JS-K)
 1600 R(M,JS-K)=SUM
      CALL FACTOR
      RETURN
      END
```

```
      SUBROUTINE OUTPUT

     0DIMENSION X(72),ALFA(72),BETA(72),GAMMA(72),DELTA(72),
     1P(5,72),T(5,72),TAU(5,5),REQ(5,5),R(5,5),TEMP(5,6),
     2C(5,72),D(5,72),CK(5,72),SP(5,6),ST(5,6),EXR(5,6),
     3TX(5),TA(10),L(20)

     0COMMON X,ALFA,BETA,GAMMA,DELTA,P,T,TAU,REQ,R,TEMP,
     1C,D,CK,SP,ST,EXR,M,NR,TX,TA,L

      PRINT 699
  699 FORMAT (1H1,////)
      PRINT 700, NR,M
  7000FORMAT(5X,8HRUN NO. ,I4,19X,30HINVENTORY PROJECTION YEAR NO. ,I1/)
      PRINT 701
  701 FORMAT(44X,9HPAY GRADE/)
      PRINT 702
  7020FORMAT(13X,3HE 3,4X,3HE 4,4X,3HE 4,4X,3HE 5,4X,3HE 5,4X, 3HE 6,4X,
     13HE 6,4X,3HE 6,4X,3HE 7,4X,3HE 7,4X, 3HE 7,4X,3HE 7)
      PRINT 703, (P(M,N), N=6,72,6)
  703 FORMAT(1H0,7X,1H1,X,12F7.1)
      PRINT 704
  704 FORMAT(5X,1HO)
      PRINT 705, (P(M,N), N=5,71,6)
  705 FORMAT(5X,1HB,2X,1H2,X,12F7.1)
      PRINT 706
  706 FORMAT(5X,1HL)
      PRINT 707, (P(M,N), N=4,70,6)
  707 FORMAT(5X,1HI,2X,1H3,X,12F7.1)
      PRINT  708
  708 FORMAT(5X,1HS)
      PRINT 709, (P(M,N), N=3,69,6)
  709 FORMAT(5X,1HE,2X,1H4,X,12F7.1)
      PRINT 710
  710 FORMAT(5X,1HR)
      PRINT 711, (P(M,N), N = 2,68,6)
```

```
711 FORMAT(5X,1HV,2X,1H5,X,12F7.1)
    PRINT 712, (P(M,N), N = 1,67,6)
712 FORMAT(1H0,7X,1H6,X,12F7.1//)
    PRINT 713
7130FORMAT(5X,25HTOTAL PROJECTED INVENTORY,10X,21HTOTAL TRAINING INPUT
   1S,10X,24HEXCESS (SHORTAGE) CF REQ/)
    PRINT 714,SP(M,1),ST(M,1),EXR(M,1)
7140FORMAT(10X,6HPG E-3,2X,F7.1,18X,6HPG E-3,2X,F7.1,18X,6HPG E-3,2X,F
   17.1)
    PRINT 715, SP(M,2), ST(M,2), EXR(M,2)
7150FORMAT(10X,6HPG E-4,2X,F7.1,18X,6HPG E-4,2X,F7.1,18X,6HPG E-4,2X,F
   17.1)
    PRINT 716, SP(M,3), ST(M,3), EXR(M,3)
7160FORMAT(10X,6HPG E-5,2X,F7.1,18X,6HPG E-5,2X,F7.1,18X,6HPG E-5,2X,F
   17.1)
    PRINT 717, SP(M,4), ST(M,4), EXR(M,4)
7170FORMAT(10X,6HPG E-6,2X,F7.1,18X,6HPG E-6,2X,F7.1,18X,6HPG E-6,2X,F
   17.1)
    PRINT 718, SP(M,5), ST(M,5), EXR(M,5)
7180FORMAT(10X,6HPG E-7,2X,F7.1,18X,6HPG E-7,2X,F7.1,18X,6HPG E-7,2X,F
   17.1/)
    PRINT 719, SP(M,6), ST(M,6), EXR(M,6)
7190FORMAT(10X,5HTOTAL,3X,F7.1,18X,5HTOTAL,3X,F7.1,18X,5HTOTAL,3X,F
   17.1//)
    PRINT 720
720 FORMAT(38X,23HTRAINING INPUT PER CELL/)
    PRINT 701
    PRINT 702
    PRINT 703, (T(M,N), N= 6,72,6)
    PRINT 704
    PRINT 705, (T(M,N), N=5,71,6)
    PRINT 706
    PRINT 707, (T(M,N), N= 4,70,6)
    PRINT 708
    PRINT 709, (T(M,N), N= 3,69,6)
```

```
      PRINT 710
      PRINT 711, (T(M,N), N= 2,68,6)
      PRINT 712, (T(M,N), N= 1,67,6)
  721 FORMAT (1H1)
      IF(M-5)730,735,730
  735 PRINT 736,NR
  736 FORMAT(1H1,////33X,31HFIVE YEAR SUMMARY  STUDY NUMBER,I2//)
      PRINT 737
      PRINT 738
  737 FORMAT(30X,4HYEAR,6X,1H1,6X,1H2,6X,1H3,6X,1H4,6X,1H5//)
  738 FORMAT (40X,24HEXCESS (SHORTAGE) OF REQ/)
      PRINT 739 (EXR(J,1), J=1,5)
      PRINT 741 (EXR(J,2), J=1,5)
      PRINT 743 (EXR(J,3), J=1,5)
      PRINT 745 (EXR(J,4), J=1,5)
      PRINT 747 (EXR(J,5), J=1,5)
      PRINT 749 (EXR(J,6), J=1,5)
  739 FORMAT (30X,6HPG E-3, 5F7.1/)
  741 FORMAT (30X,6HPG E-4, 5F7.1/)
  743 FORMAT (30X,6HPG E-5, 5F7.1/)
  745 FORMAT (30X,6HPG E-6, 5F7.1/)
  747 FORMAT (30X,6HPG E-7, 5F7.1/)
  749 FORMAT (30X,6HTOTAL ,5F7.1///)
      PRINT 732
  732 FORMAT(40X,25HTOTAL PROJECTED INVENTORY/)
      PRINT 739 (SP(J,1),J=1,5)
      PRINT 741 (SP(J,2),J=1,5)
      PRINT 743 (SP(J,3),J=1,5)
      PRINT 745 (SP(J,4),J=1,5)
      PRINT 747 (SP(J,5),J=1,5)
      PRINT 749(SP(J,6),J=1,5)
      PRINT 731
  731 FORMAT(41X,21HTOTAL TRAINING INPUTS/)
      PRINT 739 (ST(J,1),J=1,5)
      PRINT 741 (ST(J,2),J=1,5)
      PRINT 743 (ST(J,3),J=1,5)
      PRINT 745(ST(J,4),J=1,5)
```

```
      PRINT 747(ST(J,5),J=1,5)
      PRINT 749 (ST(J,6),J=1,5)
      SUMST=0.0
      DO 753 J=1,5
      DO 753 K=1,5
  753 SUMST=SUMST+ST(J,K)
      PRINT 754,SUMST
  754 FORMAT(30X,30HTOTAL FIVE YEAR TRAINING INPUT,F7.1/)
      PRINT 751, YC
  751 FORMAT(30X,20HTOTAL FIVE YEAR COST,10X,F7.1///)
      PRINT 800 (CC(J),J=1,5)
  800 FORMAT (30X,16HCOST COEFICIENTS,5F6.3//)
  730 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE TOTAL (M, TOTAL, SUM)
      DIMENSION TOTAL(5,72), SUM(5,6)
      KK = 1
      LL = 6
      DO 16 I =1,6
      SUM(M,I) = 0.0
      DO 10 J = KK,LL
   10 SUM(M,I) = SUM(M,I) + TOTAL(M,J)
      GO TO(11,12,13,14,15,16),I
   11 KK = 7
      LL = 18
      GO TO 16
   12 KK =19
      LL = 30
      GO TO 16
   13 KK = 31
      LL = 48
      GO TO 16
   14 KK = 49
      LL = 72
      GO TO 16
   15 KK = 1
      LL = 72
   16 CONTINUE
      RETURN
      END
      END
```

2. PIPE FIVE

The following listing for PIPE FIVE is written in FORTRAN 60.

```
      PROGRAM PIPE FIVE

     0DIMENSION X(72),ALFA(72),BETA(72),GAMMA(72),DELTA(72),UMB(72),
     1P(5,72),T(5,72),TAU(5,5),REQ(5,5),R(5,5),TX(5),TA(10),CC(5),
     2C(5,72),D(5,72),CK(5,72),SP(5,6),ST(5,6),EXR(5,6),
     3EA1(72),UA(72),VA(72),WA(72),A(50,100),B(50),L(20),XO(50)
     0COMMON X,ALFA,BETA,GAMMA,DELTA,P,T,TAU,REQ,R,ZZ,UMB,
     1C,D,CK,SP,ST,EXR,M,NR,L,UA,VA,WA,EA1,A,B,XO,JH,CC,TA,TX,YA,YC

      READ INPUT TAPE 5,1100,(X(N),N=1,72)
      READ INPUT TAPE 5,1100,(ALFA(N),N=1,72)
      READ INPUT TAPE 5,1100,(DELTA(N),N=6,72,6)
      READ INPUT TAPE 5,1100,(BETA(N),N=61,72)
      READ INPUT TAPE 5,1100,(GAMMA(N),N=1,72)
      READ INPUT TAPE 5,1200,((REQ(M,J),J=1,5),M=1,5)
      READ INPUT TAPE 5,1030,L(1),L(2)
      READ INPUT TAPE 5,1040,(L(I),I=3,6)
      READ INPUT TAPE 5,1040,(L(I),I=7,10)
      READ INPUT TAPE 5,1050,(L(I),I=11,16)
      READ INPUT TAPE 5,1040,(L(I),I=17,20)
      READ INPUT TAPE 5,1300,NR
      READ INPUT TAPE 5,1200,(CC(J),J=1,5)
      READ INPUT TAPE 5,1250,ZZ

 1030 FORMAT (I3,I3)
 1040 FORMAT (I3,I3,I3,I3)
 1050 FORMAT (I3,I3,I3,I3,I3,I3)
 1100 FORMAT (6F12.4)
 1200 FORMAT (5F6.0)
 1250 FORMAT (F4.0)
 1300 FORMAT (I4)
      DO 1400 N=1,72
 1400 UMB(N)=X(N)
```

```
C     PIPE FIVE MAIN PROGRAM
      CALL HCUSE FIVE
      DO 4800 M=1,5
      CALL VENT FIVE
      CALL BVECTOR
 4800 CCNTINUE
      CALL LINEAR (A,B,31,55,XC,YC)
C   31, NUMBER OF ROWS OF AMATRIX, 55 NUMBER OF COLUMNS
      DO 4850 N=1,72
 4850 X(N)=UMB(N)
      DO 4900 M = 1,5
      CALL TRAIN FIVE
      CALL VENT FIVE
      CALL TCTAL(M,T,ST)
      CALL TCTAL (M,P,SP)
      CALL EXCESS
      CALL OUTPUT
 4900 CONTINUE
      END
```

```
      SUBROUTINE HOUSE FIVE

     0DIMENSION X(72),ALFA(72),BETA(72),GAMMA(72),DELTA(72),UMB(72),
     1P(5,72),T(5,72),TAU(5,5),REQ(5,5),R(5,5),TX(5),TA(10),CC(5),
     2C(5,72),D(5,72),CK(5,72),SP(5,6),ST(5,6),EXR(5,6),
     3EA1(72),UA(72),VA(72),WA(72),A(50,100),B(50),L(20),XO(50)
     0COMMCN X,ALFA,BETA,GAMMA,DELTA,P,T,TAU,REQ,R,ZZ,UMB,
     1C,D,CK,SP,ST,EXR,M,NR,L,UA,VA,WA,EA1,A,B,XO,JH,CC,TA,TX,YA,YC
      DO 1405 N=1,72
 1405 X(N)=UMB(N)

      DO 1410 J=1,67,6
      JJ=J+4
      DO 1410 N=J,JJ
 1410 DELTA(N)=1.0

      DO 1500 N=1,60
 1500 BETA(N)=0.0

      DO 1550 M=1,5
      DO 1550 N=1,72
 1550 T(M,N)=0.0

      DO 1560 N=1,5C
 1560 XO(N) = 0.0

      DO 1570 N=1,5
      DO 1570 M=1,5
 1570 TAU(M,N) = 0.0

C       COMPUTE CONSTANTS

      DO    10 J = 2,72
   10 EA1(J) = 1.0 - (ALFA(J-1) + BETA(J-1))
      EA1(1) = 0.0
      DO    40 J = 2,72
   40 UA(J) = EA1(J) * DELTA(J-1) * (1.0 - GAMMA(J-1))
```

```
      UA(1) = 0.0
      DO   50 J = 1,7
   50 VA(J) = 0.0
      DO   60 J = 8,13
   60 VA(J) = DELTA(J-7)*GAMMA(J-7)*(1.0-( ALFA(J-7) + BETA(J-7)))
      DO   70 J = 1,13
   70 WA(J) = 0.0
      DO   80 J = 14,72
   80 WA(J)=(1.0-(ALFA(J-13)+BETA(J-13)))*DELTA(J-13)*GAMMA(J- 13)

C COMPUTE CUMMULATIVE REQUIREMENTS R(M,J)

      DO 1600 M=1,5
      JS=6
      SUM=0.0
      DO 1600 K=1,5
      SUM=SUM+REQ(M,JS-K)
 1600 R(M,JS-K)=SUM

      DO 1300 I=1,50
      DO 1300 J=1,100
 1300 A(I,J)=0.0

C   SET COST COEFFICIENTS INTO FIRST ROW OF AMATRIX

      DO 1700 I=5,25,5
      K=I-5
      DO 1700 J=1,5
 1700 A(1,J+K)=CC(J)

      CALL AMAT

      CALL FACTOR

      RETURN
      END
```

```
      SUBROUTINE VENT FIVE

     0DIMENSION X(72),ALFA(72),BETA(72),GAMMA(72),DELTA(72),UMB(72),
     1P(5,72),T(5,72),TAU(5,5),REC(5,5),R(5,5),TX(5),TA(10),CC(5),
     2C(5,72),D(5,72),CK(5,72),SP(5,6),ST(5,6),EXR(5,6),
     3EA1(72),UA(72),VA(72),WA(72),A(50,100),B(50),L(20),XO(50)
     0COMMCN X,ALFA,BETA,GAMMA,DELTA,P,T,TAU,REQ,R,ZZ,UMB,
     1C,D,CK,SP,ST,EXR,M,NR,L,UA,VA,WA,EA1,A,B,XO,JH,CC,TA,TX,YA,YC

C COMPUTE  CCNSTANT C(M,N)

      C(M,1)=0.0
      DO 1700 N=2,72
 1700 C(M,N)=X(N-1)*(1.0-(ALFA(N-1)+BETA(N-1)))

C COMPUTE CONSTANT D(M,N)

      DO 1800 N=1,72
 1800 D(M,N)=0.0
      DO 1900 N=8,13
 1900 D(M,N)=DELTA(N-7)*GAMMA(N-7)*C(M,N-6)
      DO 2000 N=20,30
 2000 D(M,N)=DELTA(N-13)*GAMMA(N-13)*C(M,N-12)
      DO 2100 N=32,43
 2100 D(M,N)=DELTA(N-13)*GAMMA(N-13)*C(M,N-12)
      DO 2200 N=49,61
 2200 D(M,N)=DELTA(N-13)*GAMMA(N-13)*C(M,N-12)

C COMPUTE CONSTANT CK(M,N)

      DO 2300 N=1,72
 2300 CK(M,N)=D(M,N)+(C(M,N)*DELTA(N-1)*(1.0-GAMMA(N-1)))
      CK(M,1)=0.0
      CK(M,7)=0.0
      CK(M,19)=0.0
      CK(M,31)=0.0
      CK(M,49)=D(M,49)
```

```
C COMPUTE PROJECTED INVENTORY

      DO 4600 N=1,72
 4600 P(M,N)=CK(M,N)+T(M,N)

      DO 4700 N=1,72
 4700 X(N)=P(M,N)
      RETURN
      END



      SUBROUTINE TRAIN FIVE

     ODIMENSION X(72),ALFA(72),BETA(72),GAMMA(72),DELTA(72),UMB(72),
     1P(5,72),T(5,72),TAU(5,5),REQ(5,5),R(5,5),TX(5),TA(10),CC(5),
     2C(5,72),D(5,72),CK(5,72),SP(5,6),ST(5,6),EXR(5,6),
     3EA1(72),UA(72),VA(72),WA(72),A(50,100),B(50),L(20),XO(50)
     OCOMMON X,ALFA,BETA,GAMMA,DELTA,P,T,TAU,REQ,R,ZZ,UMB,
     1C,D,CK,SP,ST,EXR,M,NR,L,LA,VA,WA,EA1,A,B,XO,JH,CC,TA,TX,YA,YC

C  COMPUTE THE TAUS FROM XO(JH)
      DO 440 N=1,5
      GO TO (410,412,414,416,418),M
  410 TAU(M,N) = XO(N)
      GO TO 440
  412 TAU(M,N) = XO(N+5)
      GO TO 440
  414 TAU(M,N) = XO(N+10)
      GO TO 440
  416 TAU(M,N) = XO(N+15)
      GO TO 440
  418 TAU(M,N) = XO(N+20)
  440 CONTINUE
```

```
      K=1
      JA=1
      JB=2
      DO 45 I=1,10
      IF(L(JA))10,10,15
   15 IF(L(JB))20,20,25
   20 T(M,JA)=TAU(M,K)*TX(K)
      GO TO 1C
   25 MA=L(JA)
      MB=L(JB)
      DO 30 N=MA,MB
   30 T(M,N)=TAU(M,K)*TX(K)
   10 GO TC (40,44,41,44,42,44,44,43,44,44),I
   40 K=2
      GO TO 44
   41 K=3
      GO TO 44
   42 K=4
      GC TO 44
   43 K=5
   44 JA=JA+2
   45 JB=JB+2
      RETURN
      END
```

```
      SUBROUTINE FACTOR
     0DIMENSION X(72),ALFA(72),BETA(72),GAMMA(72),DELTA(72),UMB(72),
     1P(5,72),T(5,72),TAU(5,5),REC(5,5),R(5,5),TX(5),TA(10),CC(5),
     2C(5,72),D(5,72),CK(5,72),SP(5,6),ST(5,6),EXR(5,6),
     3EA1(72),UA(72),VA(72),WA(72),A(50,100),B(50),L(20),XO(50)
     0COMMON X,ALFA,BETA,GAMMA,DELTA,P,T,TAU,REQ,R,ZZ,UMB,
     1C,D,CK,SP,ST,EXR,M,NR,L,UA,VA,WA,EA1,A,B,XO,JH,CC,TA,TX,YA,YC
      JA=1
      JB=2
      DO 30 I=1,10
      TA(I) = 0.0
      IF(L(JA))10,10,15
   15 IF(L(JB))20,20,25
   20 TA(I)=1
      GO TO 10
   25 TA(I)=L(JB)-L(JA)+1
   10 JA=JA+2
   30 JB=JB+2
      IF(TA(1))61,61,60
   60 TX(1)=1./TA(1)
   61 IF(TA(2)+TA(3))63,63,62
   62 TX(2)=1./(TA(2)+TA(3))
   63 IF(TA(4)+TA(5))65,65,64
   64 TX(3)=1./(TA(4)+TA(5))
   65 IF(TA(6)+TA(7)+TA(8))67,67,66
   66 TX(4)=1./(TA(6)+TA(7)+TA(8))
   67 IF(TA(9)+TA(10))69,69,68
   68 TX(5)=1./(TA(9)+TA(10))
   69 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE EXCESS

     0DIMENSION X(72),ALFA(72),BETA(72),GAMMA(72),DELTA(72),UMB(72),
     1P(5,72),T(5,72),TAU(5,5),REQ(5,5),R(5,5),TX(5),TA(10),CC(5),
     2C(5,72),D(5,72),CK(5,72),SP(5,6),ST(5,6),EXR(5,6),
     3EA1(72),UA(72),VA(72),WA(72),A(50,100),B(50),L(20),XO(50)
     0COMMON X,ALFA,BETA,GAMMA,DELTA,P,T,TAU,REQ,R,ZZ,UMB,
     1C,D,CK,SP,ST,EXR,M,NR,L,UA,VA,WA,EA1,A,B,XO,JH,CC,TA,TX,YA,YC

C COMPUTE EXCESS OF REQUIREMENTS BY PAY GRADE

      DO 8500 J = 1,5
      EXR(M,J) = SP(M,J) - REQ(M,J)
 8500 CONTINUE
      SUM=0.0
      DO 8600 J = 1,5
 8600 SUM = SUM + EXR(M,J)
      EXR(M,6) = SUM
 8700 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE BVECTOR
     ODIMENSION X(72),ALFA(72),BETA(72),GAMMA(72),DELTA(72),UMB(72),
     1P(5,72),T(5,72),TAU(5,5),REQ(5,5),R(5,5),TX(5),TA(10),CC(5),
     2C(5,72),D(5,72),CK(5,72),SP(5,6),ST(5,6),EXR(5,6),
     3EA1(72),UA(72),VA(72),WA(72),A(50,100),B(50),L(20),XO(50)
     OCOMMON X,ALFA,BETA,GAMMA,DELTA,P,T,TAU,REQ,R,ZZ,UMB,
     1C,D,CK,SP,ST,EXR,M,NR,L,UA,VA,WA,EA1,A,B,XO,JH,CC,TA,TX,YA,YC

      B(1) = 0.0
      MI = 1
      JR = 1
      GO TO (100,105,110,115,120),M
  100 KI=2
      NI=6
      LI=1
      GO TO 1020
  105 KI=7
      NI=11
      LI=6
      GO TO 1020
  110 KI=12
      NI=16
      LI=11
      GO TO 1020
  115 KI=17
      NI=21
      LI=16
      GO TO 1020
  120 KI=22
      NI=26
      LI=21
 1020 CONTINUE
```

```
      DO 1060 I = KI,NI
 1030 SUM  = 0.0
      DO 1035 J = MI,72
 1035 SUM = SUM + CK(M,J)
      B(I)=R(M,I-LI)-SUM
      IF(B(I)) 1040,1045,1045
 1040 B(I) = 0.0
 1045 GO TO(1051,1052,1053,1054,1055),JR
 1051 MI = 7
      JR = JR + 1
      GO TO 1060
 1052 MI = 19
      JR = JR + 1
      GO TO 1060
 1053 MI = 31
      JR = JR + 1
      GO TO 1060
 1054 MI = 49
      JR = JR + 1
      GO TO 1060
 1055 IF(L(17)+L(19))610,610,650
  610 B(LI+5)=0.0
      IF(L(11)+L(13)+L(15))620,620,650
  620 B(LI+4)=0.0
      IF(L(7)+L(9))630,630,650
  630 B(LI+3)=0.0
      IF(L(3)+L(5))640,640,650
  640 B(LI+2)=0.0
C   B VECTOR TERM CORRESPONDING TO CONTROL FACTOR RESTRAINT EQUATIONS
  650 B(26+M)= B(LI+1) + ZZ*R(M,1)
 1060 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE TOTAL (M,TOTAL,SUM)

      DIMENSION TOTAL(5,72), SUM(5,6)
      KK = 1
      LL = 6
      DO 16 I =1,6
      SUM(M,I) = 0.0
      DO 10 J = KK,LL
   10 SUM(M,I) = SUM(M,I) + TOTAL(M,J)
      GO TO(11,12,13,14,15,16),I
   11 KK = 7
      LL = 18
      GO TO 16
   12 KK =19
      LL = 30
      GO TO 16
   13 KK = 31
      LL = 48
      GO TO 16
   14 KK = 49
      LL = 72
      GO TO 16
   15 KK = 1
      LL = 72
   16 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE OUTPUT

      DIMENSION X(72),ALFA(72),BETA(72),GAMMA(72),DELTA(72),UMB(72),
     1P(5,72),T(5,72),TAU(5,5),REQ(5,5),R(5,5),TX(5),TA(10),CC(5),
     2C(5,72),D(5,72),CK(5,72),SP(5,6),ST(5,6),EXR(5,6),
     3EA1(72),UA(72),VA(72),WA(72),A(50,100),B(50),L(20),XO(50)
      COMMON X,ALFA,BETA,GAMMA,DELTA,P,T,TAU,REQ,R,ZZ,UMB,
     1C,D,CK,SP,ST,EXR,M,NR,L,UA,VA,WA,EA1,A,B,XO,JH,CC,TA,TX,YA,YC

      PRINT 699
  699 FORMAT (1H1,////)
      PRINT 700, NR,M
  700 FORMAT(5X,8HRUN NO. ,I4,19X,30HINVENTORY PROJECTION YEAR NO. ,I1/)
      PRINT 701
  701 FORMAT(44X,9HPAY GRADE/)
      PRINT 702
  702 FORMAT(13X,3HE 3,4X,3HE 4,4X,3HE 4,4X,3HE 5,4X,3HE 5,4X,3HE 6,4X,
     13HE 6,4X,3HE 6,4X,3HE 7,4X,3HE 7,4X,3HE 7,4X,3HE 7)
      PRINT 703, (P(M,N), N=6,72,6)
  703 FORMAT(1HO,7X,1H1,X,12F7.1)
      PRINT 704
  704 FORMAT(5X,1HO)
      PRINT 705, (P(M,N), N=5,71,6)
  705 FORMAT(5X,1HB,2X,1H2,X,12F7.1)
      PRINT 706
  706 FORMAT(5X,1HL)
      PRINT 707, (P(M,N), N=4,70,6)
  707 FORMAT(5X,1HI,2X,1H3,X,12F7.1)
      PRINT  708
  708 FORMAT(5X,1HS)
      PRINT 709, (P(M,N), N=3,69,6)
  709 FORMAT(5X,1HE,2X,1H4,X,12F7.1)
      PRINT 710
  710 FORMAT(5X,1HR)
      PRINT 711, (P(M,N), N = 2,68,6)
  711 FORMAT(5X,1HV,2X,1H5,X,12F7.1)
      PRINT 712, (P(M,N), N = 1,67,6)
```

```
712 FORMAT(1H0,7X,1H6,X,12F7.1//)
    PRINT 713
7130FORMAT(5X,25HTOTAL PROJECTED INVENTORY,10X,21HTOTAL TRAINING INPUT
   1S,10X,24HEXCESS (SHORTAGE) OF REQ/)
    PRINT 714,SP(M,1),ST(M,1),EXR(M,1)
7140FORMAT(10X,6HPG E-3,2X,F7.1,18X,6HPG E-3,2X,F7.1,18X,6HPG E-3,2X,F
   17.1)
    PRINT 715, SP(M,2), ST(M,2), EXR(M,2)
7150FORMAT(10X,6HPG E-4,2X,F7.1,18X,6HPG E-4,2X,F7.1,18X,6HPG E-4,2X,F
   17.1)
    PRINT 716, SP(M,3), ST(M,3), EXR(M,3)
7160FORMAT(10X,6HPG E-5,2X,F7.1,18X,6HPG E-5,2X,F7.1,18X,6HPG E-5,2X,F
   17.1)
    PRINT 717, SP(M,4), ST(M,4), EXR(M,4)
7170FORMAT(10X,6HPG E-6,2X,F7.1,18X,6HPG E-6,2X,F7.1,18X,6HPG E-6,2X,F
   17.1)
    PRINT 718, SP(M,5), ST(M,5), EXR(M,5)
7180FORMAT(10X,6HPG E-7,2X,F7.1,18X,6HPG E-7,2X,F7.1,18X,6HPG E-7,2X,F
   17.1/)
    PRINT 719, SP(M,6), ST(M,6), EXR(M,6)
7190FORMAT(10X,5HTOTAL,3X,F7.1,18X,5HTOTAL,3X,F7.1,18X,5HTOTAL,3X,F
   17.1//)
    PRINT 720
720 FORMAT(38X,23HTRAINING INPUT PER CELL/)
    PRINT 701
    PRINT 702
    PRINT 703, (T(M,N), N= 6,72,6)
    PRINT 704
    PRINT 705, (T(M,N), N=5,71,6)
    PRINT 706
    PRINT 707, (T(M,N), N= 4,70,6)
    PRINT 708
    PRINT 709, (T(M,N), N= 3,69,6)
    PRINT 710
    PRINT 711, (T(M,N), N= 2,68,6)
    PRINT 712, (T(M,N), N= 1,67,6)
```

```
721 FORMAT (1H1)
    IF(M-5)730,735,730
735 PRINT 736,NR
736 FORMAT(1H1,////33X,31HFIVE YEAR SUMMARY  STUDY NUMBER,I2//)
    PRINT 737
    PRINT 738
737 FORMAT(30X,4HYEAR,6X,1H1,6X,1H2,6X,1H3,6X,1H4,6X,1H5//)
738 FORMAT (40X,24HEXCESS (SHORTAGE) OF REQ/)
    PRINT 739 (EXR(J,1), J=1,5)
    PRINT 741 (EXR(J,2), J=1,5)
    PRINT 743 (EXR(J,3), J=1,5)
    PRINT 745 (EXR(J,4), J=1,5)
    PRINT 747 (EXR(J,5), J=1,5)
    PRINT 749 (EXR(J,6), J=1,5)
739 FORMAT (30X,6HPG E-3, 5F7.1/)
741 FORMAT (30X,6HPG E-4, 5F7.1/)
743 FORMAT (30X,6HPG E-5, 5F7.1/)
745 FORMAT (30X,6HPG E-6, 5F7.1/)
747 FORMAT (30X,6HPG E-7, 5F7.1/)
749 FORMAT (30X,6HTOTAL ,5F7.1///)
    PRINT 732
732 FORMAT(40X,25HTOTAL PROJECTED INVENTORY/)
    PRINT 739 (SP(J,1),J=1,5)
    PRINT 741 (SP(J,2),J=1,5)
    PRINT 743 (SP(J,3),J=1,5)
    PRINT 745 (SP(J,4),J=1,5)
    PRINT 747 (SP(J,5),J=1,5)
    PRINT 749(SP(J,6),J=1,5)
    PRINT 731
731 FORMAT(41X,21HTOTAL TRAINING INPUTS/)
    PRINT 739 (ST(J,1),J=1,5)
    PRINT 741 (ST(J,2),J=1,5)
    PRINT 743 (ST(J,3),J=1,5)
    PRINT 745(ST(J,4),J=1,5)
    PRINT 747(ST(J,5),J=1,5)
    PRINT 749 (ST(J,6),J=1,5)
```

```
      SUMST=0.0
      DO 753 J=1,5
      DO 753 K=1,5
  753 SUMST=SUMST+ST(J,K)
      PRINT 754,SUMST
  754 FORMAT(30X,30HTOTAL FIVE YEAR TRAINING INPUT,F7.1/)
      PRINT 751, YC
  751 FORMAT(30X,20HTOTAL FIVE YEAR COST,10X,F7.1///)
      PRINT 800 (CC(J),J=1,5)
  800 FORMAT (30X,16HCOST COEFICIENTS,5F6.3//)
  730 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE AMAT

     ODIMENSION X(72),ALFA(72),BETA(72),GAMMA(72),DELTA(72),UMB(72),
     1P(5,72),T(5,72),TAU(5,5),REG(5,5),R(5,5),TX(5),TA(10),CC(5),
     2C(5,72),D(5,72),CK(5,72),SP(5,6),ST(5,6),EXR(5,6),
     3EA1(72),UA(72),VA(72),WA(72),A(5C,100),B(50),L(2C),XO(50)
     OCOMMON X,ALFA,BETA,GAMMA,DELTA,P,T,TAU,REQ,R,ZZ,UMB,
     1C,D,CK,SP,ST,EXR,M,NR,L,UA,VA,WA,EA1,A,B,XO,JH,CC,TA,TX,YA,YC

C       FIRST YEAR ALL RATES     (UNITY MATRIX)

      IF(L(1))200,200,205
  205 A(2,1)=1.0
  200 IF(L(3)+L(5))21C,210,215
  215 A(2,2)=1.0
      A(3,2)=1.0
  210 IF(L(7)+L(9))220,220,225
  225 A(2,3)=1.0
      A(3,3)=1.0
      A(4,3)=1.0
  220 IF(L(11)+L(13)+L(15))230,23C,235
  235 A(2,4)=1.0
      A(3,4)=1.0
      A(4,4)=1.0
      A(5,4)=1.0
  230 IF(L(17)+L(19))24C,240,245
  245 A(2,5)=1.0
      A(3,5)=1.0
      A(4,5)=1.0
      A(5,5)=1.0
      A(6,5)=1.0
  240 CONTINUE
```

```
C     SECOND YEAR  E-7

      CALL SUM(UA,USUM,0,0,L(17),L(18),L(19),L(20),1,TU,72)
      CALL SUM(WA,WSUM,L(11),L(12),L(13),L(14),L(15),L(16),13,TW,61)
      A(11,4)=WSUM/TW
      A(11,5)=USUM/TU

C     SECOND YEAR E-6

      CALL SUM(WA,WSUM,0,0,L(7),L(8),L(9),L(10),13,TW,43)
      CALL SUM(UA,USUM,L(11),L(12),L(13),L(14),L(15),L(16),1,TU,48)
      A(10,3)=WSUM/TW
      A(10,4)=USUM/TU+A(11,4)
      A(10,5) = A(11,5)

C     SECOND YEAR E-5

      CALL SUM(WA,WSUM,0,0,L(3),L(4),L(5),L(6),13,TW,30)
      CALL SUM(UA,USUM,0,0,L(7),L(8),L(9),L(10),1,TU,30)
      A(9,2)=WSUM/TW
      A(9,3)=USUM/TU+A(10,3)
      A(9,4) = A(10,4)
      A(9,5) = A(11,5)

C     SECOND YEAR E-4

      CALL SUM(VA,VSUM,0,0,L(1),L(2),0,0,7,TV,13)
      CALL SUM(UA,USUM,0,0,L(3),L(4),L(5),L(6),1,TU,18)
      A(8,1)=VSUM/TV
      A(8,2)=USUM/TU+A(9,2)
      A(8,3) = A(9,3)
      A(8,4) = A(10,4)
      A(8,5) = A(11,5)
```

```
C       SECOND YEAR   E-3

        CALL  SUM(UA,USUM,0,0,L(1),L(2),0,0,1,TU,6)
        A(7,1)=USUM/TU+A(8,1)
        A(7,2) = A(8,2)
        A(7,3) = A(9,3)
        A(7,4) = A(10,4)
        A(7,5) = A(11,5)

        CALL FIX (L,2)

C       THIRD YEAR   E-7

        CALL  SUM(WA,WSUM,L(11),L(12),L(13),L(14),L(15),L(16),13,TW,61)
        CALL  SUM(UA,USUM,0,0,L(17),L(18),L(19),L(20),1,TU,72)
        CALL  SUM(UA,XSUM,L(11),L(12),L(13),L(14),L(15),L(16),13,TX,72)
        CALL  SUM(WA,ZSUM,0,0,L(7),L(8),L(9),L(10),25,TZ,61)
        A(16,5)=USUM*A(11,5)/TU
        A(16,4)=XSUM*A(11,4)/TX+WSUM*(A(10,4)-A(11,4))/TW
        A(16,3)=ZSUM*A(10,3)/TZ

C       THIRD YEAR    E-6

        CALL  SUM(WA,WSUM,0,0,L(7),L(8),L(9),L(10),13,TW,43)
        CALL  SUM(UA,USUM,L(11),L(12),L(13),L(14),L(15),L(16),1,TU,48)
        CALL  SUM(UA,YSUM,   0,0,L(7),L(8),L(9),L(10),13,TY,48)
        CALL  SUM(WA,ZSUM,0,0,L(3),L(4),L(5),L(6),25,TZ,43)
        A(15,3)=YSUM*A(10,3)/TY+WSUM*(A(9,3)-A(10,3))/TW    + A(16,3)
        A(15,2)=ZSUM*A(9,2)/TZ
        A(15,4)=USUM*(A(10,4)-A(11,4))/TU+A(16,4)
        A(15,5)=A(16,5)
```

```
C       THIRD YEAR   E-5

        CALL SUM(WA,WSUM,0,0,L(3),L(4),L(5),L(6),13,TW,30)
        CALL SUM(UA,USUM,0,0,L(7),L(8),L(9),L(10),1,TU,30)
        CALL SUM(WA,ZSUM,0,0,L(1),L(2),0,0,19,TZ,30)
        CALL SUM(UA,YSUM,0,0,L(3),L(4),L(5),L(6),13,TY,30)
        A(14,1)=ZSUM*A(8,1)/TZ
        A(14,2)=YSUM*A(9,2)/TY+WSUM*(A(8,2)-A(9,2))/TW   + A(15,2)
        A(14,3)=USUM*(A(9,3)-A(10,3))/TU+A(15,3)
        A(14,4)=A(15,4)
        A(14,5)=A(16,5)

C       THIRD YEAR E-4

        CALL SUM(VA,VSUM,0,0,L(1),L(2),0,0,7,TV,13)
        CALL SUM(UA,USUM,0,0,L(3),L(4),L(5),L(6),1,TU,18)
        CALL SUM(UA,XSUM,0,0,L(1),L(2),0,0,7,TX,18)
        A(13,1)=XSUM*A(8,1)/TX+VSUM*(A(7,1)-A(8,1))/TV+A(14,1)
        A(13,2)=USUM*(A(8,2)-A(9,2))/TU+A(14,2)
        A(13,3)=A(14,3)
        A(13,4)=A(15,4)
        A(13,5)=A(16,5)

C       THIRD YEAR   E-3

        CALL SUM(UA,USUM,0,0,L(1),L(2),0,0,1,TU,6)
        A(12,1)=USUM*(A(7,1)-A(8,1))/TU+A(13,1)
        A(12,2)=A(13,2)
        A(12,3)=A(14,3)
        A(12,4)=A(15,4)
        A(12,5)=A(16,5)

        CALL FIX (L,3)
```

```
C       FOURTH YEAR E-7
        CALL SUM(WA,WSUM,L(11),L(12),L(13),L(14),L(15),L(16),13,TW,61)
        CALL SUM(UA,USUM,0,0,L(17),L(18),L(19),L(20),1,TU,72)
        CALL SUM(UA,XSUM,L(11),L(12),L(13),L(14),L(15),L(16),13,TX,72)
        CALL SUM(WA,ZSUM,0,0,L(3),L(4),L(5),L(6),37,TZ,61)
        CALL SUM(UA,YSUM,  0,0,L(7),L(8),L(9),L(10),25,TY,72)
        CALL SUM(WA,ASUM,  0,0,L(7),L(8),L(9),L(10),25,TA,61)
        A(21,2)=ZSUM*A(15,2)/TZ
        A(21,3)=YSUM*A(16,3)/TY+ASUM*(A(15,3)-A(16,3))/TA
        A(21,4)=XSUM*A(16,4)/TX+WSUM*(A(15,4)-A(16,4))/TW
        A(21,5)=USUM*A(16,5)/TU

C       FOURTH YEAR E-6

        CALL SUM(UA,USUM,L(11),L(12),L(13),L(14),L(15),L(16),1,TU,48)
        CALL SUM(WA,WSUM,0,0,L(7),L(8),L(9),L(10),13,TW,43)
        CALL SUM(UA,YSUM,  0,0,L(7),L(8),L(9),L(10),13,TY,48)
        CALL SUM(WA,CSUM,0,0,L(3),L(4),L(5),L(6),25,TC,43)
        CALL SUM(UA,BSUM,0,0,L(3),L(4),L(5),L(6),25,TB,48)
        CALL SUM(WA,ZSUM,0,0,L(1),L(2),0,0,31,TZ,43)
        A(20,1)=ZSUM*A(14,1)/TZ
        A(20,2)=BSUM*A(15,2)/TB+CSUM*(A(14,2)-A(15,2))/TC+A(21,2)
        A(20,3)=YSUM*(A(15,3)-A(16,3))/TY+WSUM*(A(14,3)-A(15,3))/TW+
       1A(21,3)
        A(20,4)=USUM*(A(15,4)-A(16,4))/TU+A(21,4)
        A(20,5)=A(21,5)
```

```
C     FOURTH YEAR E-5

      CALL SUM(WA,WSUM,0,0,L(3),L(4),L(5),L(6),13,TW,30)
      CALL SUM(UA,USUM,C,0,L(7),L(8),L(9),L(10),1,TU,30)
      CALL SUM(UA,XSUM,0,0,L(3),L(4),L(5),L(6),13,TX,30)
      CALL SUM(WA,ZSUM,0,0,L(1),L(2),0,0,19,TZ,30)
      CALL SUM(UA,YSUM,0,0,L(1),L(2),0,0,19,TY,30)
      A(19,1)=YSUM*A(14,1)/TY+ZSUM*(A(13,1)-A(14,1))/TZ+A(20,1)
      A(19,2)=XSUM*(A(14,2)-A(15,2))/TX+WSUM*(A(13,2)-A(14,2))/TW+
     1A(20,2)
      A(19,3)=USUM*(A(14,3)-A(15,3))/TU+A(2C,3)
      A(19,4)=A(20,4)
      A(19,5)=A(21,5)

C     FOURTH YEAR E-4

      CALL SUM(UA,XSUM,C,0,L(1),L(2),0,0,7,TX,18)
      CALL SUM(UA,USUM,0,0,L(3),L(4),L(5),L(6),1,TU,18)
      CALL SUM(VA,VSUM,0,0,L(1),L(2),0,0,7,TV,13)
      A(18,1)=XSUM*(A(13,1)-A(14,1))/TX+VSUM*(A(12,1)-A(13,1))/TV+
     1A(19,1)
      A(18,2)=USUM*(A(13,2)-A(14,2))/TU+A(19,2)
      A(18,3)=A(19,3)
      A(18,4)=A(20,4)
      A(18,5)=A(21,5)

C     FOURTH YEAR E-3

      CALL SUM(UA,USUM,0,0,L(1),L(2),0,0,1,TU,6)
      A(17,1)=USUM*(A(12,1)-A(13,1))/TU+A(18,1)
      A(17,2)=A(18,2)
      A(17,3)=A(19,3)
      A(17,4)=A(20,4)
      A(17,5)=A(21,5)

      CALL FIX (L,4)
```

```
C     FIFTH YEAR E-7

      CALL SUM(WA,DSUM,0,0,L(1),L(2),0,0,43,TD,61)
      CALL SUM(WA,CSUM,0,0,L(3),L(4),L(5),L(6),37,TC,61)
      CALL SUM(UA,BSUM,0,0,L(3),L(4),L(5),L(6),37,TB,72)
      CALL SUM(WA,ZSUM,0,0,L(7),L(8),L(9),L(10),25,TZ,61)
      CALL SUM(UA,ASUM,0,0,L(7),L(8),L(9),L(10),25,TA,72)
      CALL SUM(WA,WSUM,L(11),L(12),L(13),L(14),L(15),L(16),13,TW,61)
      CALL SUM(UA,XSUM,L(11),L(12),L(13),L(14),L(15),L(16),13,TX,72)
      CALL SUM(UA,USUM,0,0,L(17),L(18),L(19),L(20),1,TU,72)
      A(26,1)=DSUM*A(20,1)/TD
      A(26,2)=BSUM*A(21,2)/TB+CSUM*(A(20,2)-A(21,2))/TC
      A(26,3)=ASUM*A(21,3)/TA+ZSUM*(A(20,3)-A(21,3))/TZ
      A(26,4)=XSUM*A(21,4)/TX+WSUM*(A(20,4)-A(21,4))/TW
      A(26,5)=USUM*A(21,5)/TU

C     FIFTH YEAR E-6

      CALL SUM(WA,ZSUM,0,0,L(1),L(2),0,0,31,TZ,43)
      CALL SUM(UA,ASUM,0,0,L(1),L(2),0,0,31,TA,48)
      CALL SUM(WA,BSUM,0,0,L(3),L(4),L(5),L(6),25,TB,43)
      CALL SUM(UA,YSUM,0,0,L(3),L(4),L(5),L(6),25,TY,48)
      CALL SUM(WA,WSUM,0,0,L(7),L(8),L(9),L(10),13,TW,43)
      CALL SUM(UA,XSUM,0,0,L(7),L(8),L(9),L(10),13,TX,48)
      CALL SUM(UA,USUM,L(11),L(12),L(13),L(14),L(15),L(16),1,TU,48)
      A(25,1)=ASUM*A(20,1)/TA+ZSUM*(A(19,1)-A(20,1))/TZ+A(26,1)
      A(25,2)=YSUM*(A(20,2)-A(21,2))/TY+BSUM*(A(19,2)-A(20,2))/TB+
     1A(26,2)
      A(25,3)=XSUM*(A(20,3)-A(21,3))/TX+WSUM*(A(19,3)-A(20,3))/TW+
     1A(26,3)
      A(25,4)=USUM*(A(20,4)-A(21,4))/TU+A(26,4)
      A(25,5)=A(26,5)
```

```
C     FIFTH YEAR E-5

      CALL SUM(WA,WSUM,0,0,L(3),L(4),L(5),L(6),13,TW,30)
      CALL SUM(UA,USUM,0,0,L(7),L(8),L(9),L(10),1,TU,30)
      CALL SUM(UA,XSUM,0,0,L(3),L(4),L(5),L(6),13,TX,30)
      CALL SUM(WA,ZSUM,0,0,L(1),L(2),0,0,19,TZ,30)
      CALL SUM(UA,ASUM,0,0,L(1),L(2),0,0,19,TA,48)
      A(24,1)=ASUM*(A(19,1)-A(20,1))/TA+ZSUM*(A(18,1)-A(19,1))/TZ+
     1A(25,1)
      A(24,2)=XSUM*(A(19,2)-A(20,2))/TX+WSUM*(A(18,2)-A(19,2))/TW+
     1A(25,2)
      A(24,3)=USUM*(A(19,3)-A(20,3))/TU+A(25,3)
      A(24,4)=A(25,4)
      A(24,5)=A(26,5)

C     FIFTH YEAR E-4

      CALL SUM(VA,VSUM,0,0,L(1),L(2),0,0,7,TV,13)
      CALL SUM(UA,USUM,0,0,L(3),L(4),L(5),L(6),1,TU,18)
      CALL SUM(UA,XSUM,0,0,L(1),L(2),0,0,7,TX,18)
      A(23,1)=XSUM*(A(18,1)-A(19,1))/TX+VSUM*(A(17,1)-A(18,1))/TV+
     1A(24,1)
      A(23,2)=USUM*(A(18,2)-A(19,2))/TU+A(24,2)
      A(23,3)=A(24,3)
      A(23,4)=A(25,4)
      A(23,5)=A(26,5)

C     FIFTH YEAR E-3

      CALL SUM(UA,USUM,0,0,L(1),L(2),0,0,1,TU,6)
      A(22,1)=USUM*(A(17,1)-A(18,1))/TU+A(23,1)
      A(22,2)=A(23,2)
      A(22,3)=A(24,3)
      A(22,4)=A(25,4)
      A(22,5)=A(26,5)

      CALL FIX (L,5)
```

```
C     DIAGCNAL ELEMENTS

      DO 500 JK=1,16,5
      JA=JK+1
      JB=JK+5
      JC=JK+4
      DO 500 N=JA,JB
      DO 500 J=JK,JC
  500 A(N+5,J+5)=A(N,J)
      DO 510 JK=1,11,5
      JA=JK+6
      JB=JK+10
      JC=JK+4
      DO 510 N=JA,JB
      DO 510 J=JK,JC
  510 A(N+5,J+5)=A(N,J)
      DO 520 JK=1,6,5
      JA=JK+11
      JB=JK+15
      JC=JK+4
      DO 520 N=JA,JB
      DO 520 J=JK,JC
  520 A(N+5,J+5)=A(N,J)
      JK=1
      JA=JK+16
      JB=JK+20
      JC=JK+4
      DO 530 N=JA,JB
      DO 530 J=JK,JC
  530 A(N+5,J+5)=A(N,J)

C     SLACKS
      K = 2
      J=26
      DO 499 I=1,25
      A(K,J) = -1.0
      K =K + 1
  499 J = J + 1
```

```
C    ADDITIONAL RESTRAINTS FOR CONTROL FACTOR (ZZ)

      DO 550 N=1,25
      A(27,N)=A(2,N)
      A(28,N)=A(7,N)
      A(29,N)=A(12,N)
      A(30,N)=A(17,N)
      A(31,N)=A(22,N)
  550 CONTINUE
      A(27,51)=1.0
      A(28,52)=1.0
      A(29,53)=1.0
      A(30,54)=1.0
      A(31,55)=1.0

      IF(L(17)+L(19))610,610,650
  610 DO 615  M=6,26,5
      DO 615 N=1,50
  615 A(M,N)=0.0
      IF(L(11)+L(13)+L(15))620,620,650
  620 DO 625 M=5,25,5
      DO 625 N=1,50
  625 A(M,N)=0.0
      IF(L(7)+L(9))630,630,650
  630 DO 635 M=4,24,5
      DO 635 N=1,50
  635 A(M,N)=0.0
      IF(L(3)+L(5))640,640,650
  640 DO 645 M=3,23,5
      DO 645 N=1,50
  645 A(M,N)=C.0
  650 CONTINUE

      RETURN
      END
```

```
      SUBROUTINE FIX(L,K)

      DIMENSION L(20),LL(20)
      IF(K-2)10,30,10
   30 DO 35 N=1,20
   35 LL(N)=L(N)
   10 DO 50 I=1,20
      IF(L(I))50,50,20
   20 L(I)=L(I)+1
   50 CONTINUE
      IF(K-5)70,60,70
   60 DO 65 N=1,20
   65 L(N)=LL(N)
   70 CONTINUE
      RETURN
      END


      SUBROUTINE  SUM(WA,WSUM,ML,MU,LL,LU,KL,KU,JT,TW,IL)

      DIMENSION WA(72)
      WSUM=0.0
      TW1 = .0
      TW2 = .0
      TW3=.0

      IF(ML)110,110,80
   80 IF(JT-13)90,82,90
   82 IF(MU)76,76,77
   76 TW3=1.0
      GO TO 72
   77 TW3=MU-ML+1
   73 IF(MU-36)110,75,75
   75 JU=MU+13
```

```
      IF(ML-36)70,71,71
   70 JL=49
      GO TO 74
   71 JL=ML+13
   74 IF(JU-JL) 88,88,84
   84 DO 85 I=JL,JU
   85 WSUM=WSUM+WA(I)
      GO TC 110
   72 IF(ML-36) 87,86,86
   87 GO TO 110
   86 JL=ML+13
   88 WSUM=WSUM+WA(JL)
      GO TC 110

   90 JL=ML+1
      IF(MU)92,92,93
   92 WSUM=WSUM+WA(JL)
      TW3=1
      GO TO 110
   93 JU=MU+1
      DO 95 I=JL,JU
   95 WSUM=WSUM+WA(I)
      TW3=JU-JL+1

  110 IF(LL)120,120,121
  121 JL=LL+JT
      IF(IL - JL)120,300,300
  300 IF(LU) 122,122,123
  122 WSUM = WSUM + WA(JL)
      TW1 = 1
      GO TC 120
```

```
123 JU=LU+JT
    TW1 = JU - JL + 1
    IF(IL - JU)310,305,305
310 JU = IL
305 IF(JU - JL)120,334,335
334 WSUM = WSUM + WA(JL)
    GO TO 120
335 DO 125 I=JL,JU
125 WSUM=WSUM+WA(I)

120 IF(KL)130,130,131
131 JL=KL+JT
350 IF(KU)132,132,133
132 WSUM=WSUM+WA(JL)
    TW2 = 1
    GO TO 130
133 JU=KU+JT
    TW2 = JU - JL + 1
    IF(IL - JU)360,355,355
360 JU = IL
355 IF(JU - JL)130,384,385
384 WSUM = WSUM + WA(JL)
    GO TO 130
385 DO 135 I=JL,JU
135 WSUM=WSUM+WA(I)

130 TW=TW1+TW2+TW3
    RETURN
    END
```

```
      SUBROUTINE LINEAR (A,B,MROW,NCCL,XO,YC)

      DIMENSION A(50,100),B(50),INFIX(10),TOL(10),KOUT(10),ERR(10)
      DIMENSION KB(100), X(50), JH(50), P(5C),Y(50),E(5C,50)
      DIMENSION Z(100), RUN(9),XO(50)
      INFIX(4) = MRCW
      INFIX(2) = NCOL
      INFIX(1) = 4
      INFIX(3) = 50
      INFIX(5) = 2
      INFIX(6) = 1
      INFIX(7) = 100
      INFIX(8) = 0
      TOL(1) = 1.E-7
      TOL(2) = 1.E-5
      TOL(3) = -1.E-6
      TOL(4) = 1.E-10
      PRM = 0.
C    FOLLOWING CARCS PRODUCE PRINTCUT OF INPUT TO LINEAR
C     WRITE OUTPUT TAPE 3, 29, (RLN(I), I=1, 6)
C     WRITE OUTPUT TAPE 3, 100, (A(1,J), J=1, NCOL)
C     WRITE OUTPUT TAPE 3, 41
C     DO 1C5 I = 2, MROW
C     WRITE OUTPUT TAPE 3, 100, (A(I,J), J=1, NCOL), B(I)
C 105 CONTINUE
C     WRITE OUTPUT TAPE 3, 1C2
 28   FORMAT(5HORUN , 6A8/1H )
 29   FORMAT(5H1RUN , 6A8/1H )
 100  FORMAT(8E13.6)
 102  FORMAT(1HO)
      CALL SIMPLX(INFIX,A,B,TOL,PRM,KCUT,ERR,JH,X,P,Y,KB,E)
C     WRITE OUTPUT TAPE 3, 28, (RUN(I), I=1, 6)
C     WRITE OUTPUT TAPE 3,30
 30   FORMAT( 35X,    1HY,13X,1HP,13X,1HX,8X,2HJH,3X,2HKB,6X,3HERR,7X,1HK
     1/1H )
      IYEAR = 25
```

```
      YC=Y(1)
      DO 33 I = 1,NCOL
      II = KB(I)
      IF(II) 31,31,32
 31   Z(I) = 0.
      GO TO 33
 32   Z(I) = X(II)
 33   CONTINUE
      NDE = 7
      IF(NDE - NCOL) 35,36,36
 35   NDE = NCOL
 36   IF( NDE - MROW) 37,40,40
 37   NDE = MROW
 40   DO 60 IY = 1,NDE
      JHH = JH(IY)
      IF(JHH) 7734,7734,7735
 7735 IF (JHH - IYEAR) 7736,7736,7734
 7736 XO(JHH)=X(IY)
 7734 CONTINUE
C   FOLLOWING CARDS PRODUCE DETAILED PRINTOUT OF LINEAR OUTPUT
C     WRITE OUTPUT TAPE 3,41
 41   FORMAT(1H )
C     IF(IY - MROW) 42,42,44
C42   WRITE OUTPUT TAPE 3,43,          Y(IY),P(IY),X(IY),JH(IY)
C43   FORMAT(1H+  18X              ,E23.6,2E14.6,I4)
C44   IF(IY- NCOL) 45,45,48
C45   WRITE OUTPUT TAPE 3,46,IY,Z(IY), KB(IY)
C46   FORMAT(2H+Z,I2,2H =, E13.6,    56X, I4)
C48   IF(IY - 4) 49,49,52
C49   WRITE OUTPUT TAPE 3,50,ERR(IY)
C50   FORMAT(1H+,78X,E14.6)
C52   IF(IY - 7) 53,53,60
C53   WRITE OUTPUT TAPE 3,54,KCUT(IY)
C54   FORMAT(1H+,92X,I3)
   60 CONTINUE
      KK = KOUT(1)
      IF (KK - 4) 2,70,62
 62   IF(KK - 7) 63,72,2
```

```
 70   WRITE OUTPUT TAPE 3,71
 71   FORMAT(21HONO FEASIBLE SCLUTION)
      GO TO 2
 63   IF(KK - 5) 2,64,67
 64   WRITE OUTPUT TAPE 3,65
 65   FORMAT(28HONO PIVOT, INFINITE SOLUTION)
      GO TO 2
 67   WRITE OUTPUT TAPE 3,68
 68   FORMAT(26HOITTERATION LIMIT EXCEEDED)
C    ITTERATION LIMIT CONTROLLEC BY INFIX(7)
      GO TC 2  .
 72   WRITE OUTPUT TAPE 3,73
 73   FORMAT(23HOILLEGAL INPUT QUANTITY)
      GO TO 2
    2 CONTINUE
      END


      SUBRCUTINE SIMPLX (INFIX,A,B,TCL,PRM,KCUT,ERR,JH,X,P,Y,KB,E)

      DIMENSION INFIX(8),A(1),B(1),TOL(4),KCUT(7),ERR(8),JH(1),X(1),
     1 P(1),Y(1),KB(1),E(1),ZZ(3), IOFIX(16) , TERR(8)
      EQUIVALENCE  (INFLAG,IOFIX(1) ), (NZ, IOFIX(2) ) ,
     1              (ME,ICFIX(3) ), (MZ,IOFIX(4)), (MF,IOFIX(5)),
     2  (MC, IOFIX(6) ),  ( NCUT, IOFIX(7) ) , ( NVER, IOFIX(8) ),
     3 ( KZ,IOFIX(9) ), (ITER, ICFIX(10) ), (INVC , ICFIX(11) ) ,
     4 (NUMVR, IOFIX(12) ), ( NUMPV, IOFIX(13) ) ,
     5  (INFS, IOFIX(14)  ) , ( JT,  IOFIX(15) ) ,( LA  , IOFIX(16) ),
     6 (ZZ(1),TPIV), (ZZ(2),TZERO),(ZZ(3),TCCST)
      DO 1340  I= 1, 8
       TERR(I) = 0.0
      IOFIX(I+8)  =  0
 1340  IOFIX(I)  =  INFIX(I)
      N = NZ
```

```
      M = MZ
      K = KZ
      LA   =   0
      DO 1308  I = 1 , 3
 1308 ZZ(I)  =  TOL(I)
      TCOST  =  -  ABSF (TCOST)
      PMIX = PRM
      M2 = M*M
      INFS = 1
      IF (N)  1304, 1304, 1371
 1371  IF (M - MF ) 1304, 1304, 1372
 1372  IF (MF - MC)  1304, 1304, 1373
 1373  IF ( MC ) 1304 , 1304,  1374
 1374  IF (ME - M ) 1304, 1375, 1375
 1375 IF( XMODF(INFLAG, 4 )  - 1 )  1400,  1320,  100
 1400   DO 1401 I = 1, M
 1401 JH(I) = 0
       KT   =  0.
        DO 1402  J = 1, N
      KB(J) = 0
      MM   =  KT + MF
      LL   =  KT + M
      KQ   = 0
      DO   1403 L = MM , LL
      IF (A(L)) 1404, 1403, 1404
 1404 KQ = KQ+1
      LQ = L
 1403   CONTINUE
            IF (KQ - 1) 1402, 1405, 1402
 1405 IA = LQ- KT
          IF ( JH(IA) ) 1402, 1406, 1402
 1406 IF (A(LQ)*B(IA)) 1402, 1407, 1407
 1407 JH(IA) = J
      KB(J)  = IA
 1402 KT = KT + ME
 1320 CONTINUE
```

```
1100 ASSIGN  1102  TC KPIV
     ASSIGN  1114  TC KJMY
     IF (LA)  1121, 1121, 1122
1121 INVC = 0
1122 NUMVR   =   NUMVR  +1
      DO 1101   I = 1, M2
1101 E(I)=0.
     MM=1
      DO 1113   I = 1, M
     E(MM) =1.0
     X(I) = B(I)
1113 MM = MM + M + 1
       DC 1110   I = MF, M
     IF (JH(I)) 1111, 1110, 1111
1111 JH(I) = 12345
1110 CONTINUE
     INFS = 1
       DC 1102  JT= 1, N
     IF ( KB(JT))  600 , 1102 , 600
1114 TY = 0.
       DC 1104  I = MF, M
     IF (JH(I)  - 12345 )  1104, 1105, 1104
1105 IF ( ABSF( Y(I) ) -  TY )  1104, 1104, 1106
1106 IR    = I
     TY =  ABSF ( Y(I) )
1104   CCNTINUE
     IF  (TY - TPIV )    1107, 1108, 1108
1107 KB(JT)= 0
     GO TC 1102
1108 JH(IR) = JT
     KB(JT) = IR
     GO TO 900
1102     CONTINUE
       DO 1109  I = 1, M
     IF ( JH(I) - 12345  )  1109, 1112, 1109
1112 JH(I) = 0
```

```
1109   CONTINUE
 100 ASSIGN   705  TC  NDEL
     ASSIGN  1000  TC  KJMY
     ASSIGN   221  TC  KPIV
1200   JIN  =  0
     NEG = 0
     DO 1201 I = MF, M
     IF ( ABSF ( X(I) )  - TZERO)  1202, 1203, 1203
1202 X(I) = 0.0
     GO TC 1201
1203 IF ( X(I) )  1208, 1201, 1205
1205 IF ( JH(I) )  1201, 1206, 1201
1208 NEG = 1
1206   JIN = 1
1201   CONTINUE
     IF (INFS - JIN ) 1320,  500,  200
 200 INFS = 0
 201 PMIX  =  0.0
 500 MM  = MC
 502 DO 503 J = 1, M
     P(J) = E(MM )
 503 MM  =MM  + M
     IF ( INFS )  501, 599, 501
 501 DO 504  J = 1, M
 504 P(J) = P(J)* PMIX
     DO 505  I = MF, M
     MM  =I
     IF  ( X(I) )  506, 507, 507
 506 DO 508  J = 1, M
     P(J) =  P(J) + E(MM )
 508 MM  = MM  + M
     GO TO 505
 507 IF (JH(I))  505, 509, 505
 509 DO 510  J = 1, M
     P(J) = P(J) - E(MM )
 510 MM  =  MM  +M
```

```
  505   CONTINUE
  599 CONTINUE
  700 JT = 0
      BB = TCOST
  701 DO  702  JM = 1, N
  703 IF ( KB(JM) )  702, 300, 702
  705 IF  ( DT - BB )  708, 702, 702
  708 BB = DT
      JT = JM
  702    CCNTINUE
        IF  (JT)    203, 203, 600
  203 K = 3 + INFS
      KZ = K
      GO TC 257
  600 DO 610  I= 1,M
  610 Y(I) =0.
      LP  =  JT*ME - ME
      LL = 0
      DO 605   I= 1,M
      LP = LP + 1
      IF  (A(LP))   601, 602, 601
  601 DO 606  J  =  1,M
      LL = LL + 1
  606 Y(J) = Y(J) + A(LP) * E(LL)
      GO TC 605
  602 LL = LL + M
  605 CONTINUE
  699 GO TO KJMY , ( 1000 , 1114 ,  1392  )
 1000 IR = 0
      AA = 0.0
      IA  =  0
      DO 1050  I = MF,  M
      IF ( X(I) ). 1050, 1041, 1050
 1041 YI  =  ABSF ( Y(I) )
      IF  (  YI  -  TPIV   )   1050, 1050, 1C42
 1042 IF  ( JH(I) )  1043, 1044, 1043
```

```
1043 IF  (IA)    1050, 1048, 1C50
1048  IF  ( Y(I) )  1050, 105C, 1045
1044 IF   (IA)   1045, 1046, 1045
1045 IF   (  YI     -  AA    )    1C50, 1050, 1047
1046    IA = 1
1047 AA   =  YI
     IR   =  I
1050 CONTINUE
     IF (IR)1099,1001,1099
1001 AA = 1.0E+20
     DO 1010 IT = MF , M
     IF  ( Y(IT)  -  TPIV )  101C, 1010, 1CC2
1002 IF  ( X(IT) ) 1010, 1010, 1C03
1003 XY = X(IT) / Y(IT)
     IF ( XY - AA )  1004, 10C5, 1010
1005 IF ( JH(IT))  1010, 10C4, 1C10
1004 AA = XY
     IR = IT
1010 CONTINUE
     IF (NEG)   1016, 1099, 1C16
1016 BB = - TPIV
     DO 1C30 I  = MF , M
     IF (X(I))   1012, 1030, 103C
1012  IF  ( Y(I)    -  BB  )   1C22, 1030, 1C30
1022 IF  (  Y(I) * AA  -  X(I)   )   1024, 1C24, 103C
1024 BB  =  Y(I)
     IR  =  I
1030 CONTINUE
1099 CONTINUE
 206   IF( IR )   207, 2C7, 21C
     KZ = K
 207 K = 5
 257 IF (PMIX)   201, 400, 201
 210 IF (ITER -NCUT )    900,  1C0,  160
 900 NUMPV  =  NUMPV  +  1
     YI = -Y(IR)
```

```
      Y(IR) = -1.
      LL = 0
  903 DO 904  L =  IR, M2, M
      IF  ( E(L) )  905, 914, 905
  914 LL = LL + M
      GO TC 904
  905 XY =  E(L) / YI
      E(L) =0.
      DO 906  I  = 1, M
      LL= LL +1
  906 E(LL) = E(LL) +XY* Y(I)
  904 CONTINUE
       XY = X(IR) / YI
      X(IR) = 0.
      DO 908  I  = 1, M
  908 X(I) = X(I) +XY* Y(I)
      Y(IR) = -YI
  999  GO TC KPIV , ( 221, 1102 )
  221  IA   = JH(IR)
      IF ( IA ) 213, 213, 214
  214 KB( IA ) = 0
  213 KB(JT)   = IR
      JH(IR)   = JT
      LA  =  0
      ITER   = ITER   +1
      INVC   = INVC   +1
      IF (INVC - NVER  ) 1200, 1320,1200
  160 K = 6
      KZ = K
  400  ASSIGN 410 TC NDEL
      DO 401 I = 1, M
  401 Y(I) =-B(I)
      DO 402 I = 1, M
      JA = JH(I)
      IF (JA) 403, 402, 403
  403 IA =ME* (JA-1)
```

```
      DO 405 IT = 1, M
      IA = IA + 1
      IF(A(IA) )  415, 405, 415
  415 Y(IT) =Y(IT) +X(I) * A(IA)
  405 CONTINUE
  402 CONTINUE
      DO 481  I = 1, M
      YI  = Y(I)
      IF ( JH(I) )  472, 471, 472
  471  YI = YI + X(I)
  472 TERR(LA+1) = TERR(LA+1) + ABSF(YI)
      IF  ( ABSF (TERR(LA+2))- ABSF (  YI   )  )   482, 481, 481
  482 TERR(LA+2) = YI
  481   CONTINUE
      DO 411  I = 1, M
      JM = JH(I)
      IF ( JM     )  300 , 411 , 300
  410 TERR(LA+3)  =  TERR(LA +3)  +  ABSF( DT)
      IF (ABSF(TERR(LA+4))   -   ABSF(DT)  )  413, 411, 411
  413  TERR(LA+4)  =  DT
  411 CONTINUE
      IF (LA)   193, 191,  193
  191 LA  =  4
      IF (INFLAG  -  4  )  1320,  193,  193
  193 IF  (K-5)   1392,  194,  1392
  194 ASSIGN 1392 TO KJMY
      GO TO 600
 1304 K  =  7
      KZ = K
 1392   DO  1309  I=  1, 8
 1309 ERR(I)           =   TERR(I)
       DO 1329  I = 1, 7
 1329 KOUT(I)  =  ICFIX(I+8)
      RETURN
  300 DT = 0.
```

```
     LL   = (JM - 1) * ME
301 DO  303    MM  = 1, M
    LL    =  LL  + 1
    IF ( A( LL ))304, 303, 3C4
304 DT = DT + P( MM ) * A ( LL )
303 CONTINUE
399   GO TO NDEL , ( 410 , 7C5 )
     END
    END
```